WORKED PROBLEMS IN HEAT AND HEAT ENGINES

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WITH EXERCISES AND EXAMINATION QUESTIONS

BY

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The student is expected to use this book after he has studied each section of the subject in his text-book. He should first make a serious attempt to solve each question for himself before reading the solution here given, then study the solution, and finally work through the exercises at the end of the chapter.

Incidentally it may be mentioned that in the worked examples a fuller and more explanatory style is often used than it is necessary for the student to adopt either in his exercises or in the examination, for in a book of this kind fullness of explanation is always an error on the right side if an error at all.

R. W. HUTCHINSON.

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EXAMINATIONS.

Candidates for the Matriculation Examination and examinations of a similar standard should study Chapters I.-IX., omitting some of the more difficult worked examples and exercises if necessary. Inter. B.Sc. candidates should study thoroughly Chapters I.-X. and the simpler portions of Chapter XI., whilst B.Sc. candidates will require the whole of Chapters I.-XI. Engineering students should work steadily through the whole book: they will be all the better for the extra "theory" they will encounter in their reading.

CONTENTS.

WORKED PROBLEMS IN HEAT AND HEAT-ENGINES.

CHAPTER I.

THERMOMETRY.

Important Note.—In studying any formula the following plan should be adopted:—(1) **Thoroughly master the method by which the relation is obtained** (this is **absolutely essential**). (2) Note carefully the conditions under which the formula is applicable. (3) If the formula is fundamental, *learn it*. In all cases, however, the method of work is of more importance than the formulated result.

Conversion of Thermometer Scales.—It is often necessary to convert temperatures expressed on one scale into the corresponding temperatures on one of the other scales. In doing so there are two things to be noticed: (1) Since the interval of temperature between the freezing and boiling points is constant, it follows that 180 Fahrenheit degrees = 100 Centigrade degrees = 80 Réaumur degrees. (2) The zero of the Fahrenheit scale is 32 degrees below freezing point—*i.e.* 40° F. indicates a temperature 8 degrees F. above freezing point. If, therefore, F, C, and R denote corresponding readings on the Fahrenheit, Centigrade, and Réaumur scales respectively, we have that (F − 32), C and R denote, in each case, the number of degrees the given temperature is above freezing point. Hence, from (1) above—

$$(F - 32) : C : R :: 180 : 100 : 80.$$

That is—
$$(F - 32) : C : R :: 9 : 5 : 4.$$

$$\therefore \frac{(F - 32)}{9} = \frac{C}{5} = \frac{R}{4}.$$

Clearly a degree Fahrenheit is $\frac{5}{9}$ of a degree Centigrade, and a degree Centigrade is $\frac{4}{5}$ of a degree Réaumur.

CHAPTER II.

EXPANSION OF SOLIDS.

Approximations.—The following brief notes on "small quantities" and "approximations" will be of service in dealing with problems on expansion.

(1) The square or cube (or higher power) of any *small* quantity is *very small indeed* in comparison with that small quantity: thus $(\cdot 0002)^2 = \cdot 00000004$, and $(\cdot 0002)^3 = \cdot 000000000008$, the products in each case being very small indeed in comparison with $\cdot 0002$.

(2) Similarly, the product of any two very small quantities is *very small indeed* in comparison with either of those quantities: this is readily seen in the example, $\cdot 00002 \times \cdot 00003 = \cdot 0000000006$.

(3) It is evident, therefore, that if two quantities x and y are themselves very small in comparison with a third quantity A, then x^2, y^2 and xy will be **negligibly small** compared with the same quantity A.

(4) Now suppose *a and b to be small quantities compared with unity*: then:—

$$(\mathbf{1} + a)^2 = 1 + 2a + a^2 = \mathbf{1 + 2}a$$
$$(\mathbf{1} - a)^2 = 1 - 2a + a^2 = \mathbf{1 - 2}a$$

for a^2 is negligible by (1) above. Similarly:—

$$(\mathbf{1} + a)^3 = 1 + 3a + 3a^2 + a^3 = \mathbf{1 + 3}a$$
$$(\mathbf{1} - a)^3 = 1 - 3a + 3a^2 - a^3 = \mathbf{1 - 3}a.$$

Also:—

$$(\mathbf{1} + a)(\mathbf{1} + b) = 1 + a + b + ab = \mathbf{1} + a + b.$$
$$(\mathbf{1} - a)(\mathbf{1} - b) = 1 - a - b + ab = \mathbf{1} - a - b.$$

since the product ab is negligible. Further:—

$$\frac{1}{1+a} = 1 - a + a^2 - a^3 + \text{etc.} = \mathbf{1 - a}$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \text{etc.} = \mathbf{1 + a}.$$

Finally:

$$\frac{1+a}{1+b} = 1 + a - b - ab + \text{etc.} = \mathbf{1 + a - b},$$

and similarly:— $\dfrac{1-a}{1-b} = \mathbf{1 - a + b}.$

Limiting Value of a Ratio.—(1) Consider the ratio $\frac{a}{x}$. Now if a is a constant quantity, the fraction $\frac{a}{x}$ can be made as small as we please by sufficiently increasing x: thus if $a=6$, then for $x=1$ we have $a/x=6/1=6$; for $x=1{,}000{,}000$ we have $a/x = 6/1{,}000{,}000 = {\cdot}000006$ and so on. Hence if x be made large enough we can reduce the value of a/x to a quantity less than any assignable quantity. This is expressed by saying that *the limiting value, or the limit of* $\frac{a}{x}$ *is zero when* x *is infinitely great.* Similarly, the smaller x is, the greater $\frac{a}{x}$ becomes, and *the limit of* $\frac{a}{x}$ *is infinite when* x *is zero.*

(2) Now take the ratio $\frac{ax^2 + bx}{x}$, and consider what will be the limit of this ratio when $x = 0$. If we substitute $x = 0$ directly, the fraction reduces to $0/0$, which may mean anything; but the ratio is evidently equal to $ax + b$, and, when $x = 0$, this becomes b; i.e. b is the limiting value of $\frac{ax^2 + bx}{x}$ when $x = 0$. Thus, *although both numerator and denominator reduce to zero for* $x = 0$, *the ratio has a definite limiting value,* b, *for this value of* x.

Formulae for Calculations on Expansion of Solids.—The calculations considered will only be those which involve the *mean coefficients of expansion.*

It should be noticed that, in defining the mean coefficient of expansion we have to consider the ratio of the mean expansion, per degree rise of temperature, **to the original length or area or volume, as the case may be, at 0° C.** Thus, if the length of a rod be L_t at $t°$ C., and L_{t_1} at a higher temperature $t_1°$ C., the mean expansion per degree is $\frac{L_{t_1} - L_t}{t_1 - t}$, and the mean coefficient of linear expansion (l) between $t°$ and $t_1°$ is given by $l = \frac{L_{t_1} - L_t}{L_0 (t_1 - t)}$, *where* L_0 *is the length at* 0° *C.*

If L_0 be the length of a body at 0° C. and L_t its length at $t°$ C., and if l be the coefficient of linear expansion :—

$$L_t = L_0 (1 + lt) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

It is useful to have a relation between L_t, the length of a body at $t°$ C., and L_{t_1}, its length at $t_1°$ C. This is established as follows :—

$$\frac{L_{t_1}}{L_t} = \frac{L_0 (1 + lt_1)}{L_0 (1 + lt)} = \frac{1 + lt_1}{1 + lt}$$

$$\therefore \ L_{t_1} = L_t \frac{1 + lt_1}{1 + lt} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

If l is very small and the difference between the two temperatures t_1 and t is not too large, (2) may be expressed approximately thus :—

$$L_{t_1} = L_t (1 + lt_1 - lt)$$

i.e.

$$L_{t_1} = L_t \{1 + l (t_1 - t)\} \quad \ldots\ldots\ldots\ldots \quad (3)$$

This approximate expression (3) may often be used with problems on solids, and sometimes with problems on liquids, **but with gases the expansions are too great to admit of its use in problems or experiments.**

For convenience, the corresponding expressions for surface and volume expansion are given below :—

$$S_t = S_0 (1 + st) \ \ldots\ldots\ldots \ (4) \qquad V_t = V_0 (1 + ct) \ldots\ldots\ldots \ (7)$$

$$S_{t_1} = S_t \frac{1 + st_1}{1 + st} \ \ldots\ldots\ldots \ (5) \qquad V_{t_1} = V_t \frac{1 + ct_1}{1 + ct} \ \ldots\ldots\ldots (8)$$

$$S_{t_1} = S_t \{1 + s (t_1 - t)\} \ \ldots \ (6) \qquad V_{t_1} = V_t \{1 + c (t_1 - t)\} \ldots \ (9)$$

where s = coefficient of surface expansion, and c = coefficient of volume expansion ($s = 2l$ and $c = 3l$ approx.).

Notice that if, for example, ·000017 is the coefficient of linear expansion of copper, then 1 cm. of copper at 0° C. increases by ·000017 cm. when raised to 1° C., 1 ft. increases by ·000017 ft., 1 yard by ·000017 yard, and so on.

A coefficient per degree Fahrenheit is five-ninths of the coefficient per degree Centigrade.

Many problems can be worked from first principles quite readily instead of by formula: this method is often preferable.

Worked Examples.—(1) *The length of an iron rod at* 0° *C. is* 100 *cm. Find its length at* 10° *C., the mean coefficient of linear expansion of iron being* ·000012.

1 cm. *increases by* ·000012 cm. for 1° C. rise,

∴ 1 cm. „ „ ·000012 × 10 cm. for 10° C. rise,

∴ 100 cm. „ „ ·000012 × 10 × 100 cm. for 10° C. rise.

$$\therefore \text{ total length} = 100 + (\cdot 000012 \times 10 \times 100),$$

i.e.
$$L_{10} = 100 + \cdot 012 = 100\cdot 012 \text{ cm}$$

or by formula:—

$$L_t = L_0 (1 + lt)$$
$$\therefore L_{10} = 100 (1 + \cdot 000012 \times 10)$$
$$= 100\cdot 012 \text{ cm.}$$

(2) *The volume of a piece of glass at* 100° *C. is* 100·258 *c.c., and its volume at* 0° *C. is* 100 *c.c. Find the mean coefficient of cubical expansion of glass between* 0° *C. and* 100° *C., and thence deduce approximately the mean coefficient of linear expansion between the same limits of temperature.*

$$V_t = V_o (1 + ct).$$
$$\therefore c = \frac{V_t - V_o}{V_o t}.$$

Here then—

$$c = \frac{100\cdot 258 - 100}{100 \times 100} = \frac{\cdot 258}{10000} = \cdot 0000258.$$

When l is small, $c = 3l$ or $l = c/3$. Hence, the mean coefficient of linear expansion is ·0000258/3 = ·0000086.

(3) *The length of a copper rod at* 10° C. *is* 200·034 *cm. Find its length at* 100° *C., the mean coefficient of linear expansion of copper being* ·000017.

Applying formula (3) we have—

$$L_{100} = L_{10}(1 + 90l).$$

$$\therefore\ L_{100} = 200{\cdot}034\,[1 + (90 \times {\cdot}000017)]\ \text{cm.}$$
$$= 200{\cdot}034\,(1{\cdot}00153)\ \text{cm.}$$
$$= 200{\cdot}34\ \text{cm.}$$

(4) *A brass rod is found to measure* 100·019 *cm. at* 10° *C., and* 100·19 *cm. at* 100° *C. Find the mean coefficient of linear expansion of brass between* 10° *C. and* 100° *C.*

$$L_{100} = L_{10}(1 + 90l).$$

$$\therefore\ l = \frac{L_{100} - L_{10}}{L_{10} \times 90} = \frac{100{\cdot}19 - 100{\cdot}019}{100{\cdot}019 \times 90}$$

i.e. $$l = \frac{0{\cdot}171}{100{\cdot}019 \times 90} = {\cdot}0000189.$$

The above is the approximation. If we work the question by the accurate formula, we shall find the approximation is a very close one.

(5) *A steel metre scale measures* 100·0165 *cm. at* 15° *C.; at what temperature does it measure exactly one metre? The mean coefficient of linear expansion of steel is* ·000011.

Employing formula (3), let t denote the required temperature. Then—

$$L_{t_1} = L_t\,[1 + l\,(t_1 - t)]$$
$$100{\cdot}0165 = 100\,[1 + {\cdot}000011\,(15 - t)]$$
$$100{\cdot}0165 = 100 + {\cdot}0011\,(15 - t),$$
$$\therefore\ 15 - t = \frac{{\cdot}0165}{{\cdot}0011} = 15.$$
$$\therefore\ t = 0.$$

That is, the scale is correct at 0° C.

(6) *A glass rod is* 1 *metre long at* 0° *C.; find its length at* − 10° *C., the mean coefficient of linear expansion of glass being* ·0000086.

The mean coefficient of expansion is also the mean co-

efficient of contraction, but if we retain the sign of t in the formula—

$$L_t = L_0 (1 + lt)$$

it is applicable to all cases of expansion *or* contraction.

Thus—

$$L_{-10} = L_0 [1 + l(-10)]$$
$$= L_0 (1 - 10l).$$
$$\therefore L_{-10} = 1 [1 - 10 (\cdot 0000086)]$$
$$= 1 [1 - \cdot 000086]$$
$$= \cdot 999914 \text{ metre}$$
$$= 99 \cdot 9914 \text{ cm.}$$

(7) *Find the increase in the capacity of a cylindrical steam engine boiler which, at* 0° *C., is* 5 *metres long and* 3 *metres diameter, when heated from* 10° *C. to* 100° *C., the linear coefficient of iron being* ·0000123.

Note that if a hollow vessel be heated the increase in volume is found in the same way as if the whole of the interior were composed of the same material as the vessel: a little consideration will convince the student of the truth of this.

Coefficient of volume expansion of iron $= \cdot 0000123 \times 3$
$= \cdot 0000369$

Volume of boiler at 0° = area of end × length
$= \pi (1 \cdot 5)^2 \times 5 = 11 \cdot 25\pi$ c.m.

Volume of boiler at 10° $= 11 \cdot 25\pi (1 + \cdot 0000369 \times 10)$ c.m.

Volume of boiler at 100° $= 11 \cdot 25\pi (1 + \cdot 0000369 \times 100)$ c.m.

$\therefore$ Increase in volume $= V_{100} - V_{10}$
$= 11 \cdot 25\pi (1 \cdot 00369 - 1 \cdot 000369)$ c.m.
$= (11 \cdot 25\pi \times \cdot 003321)$ c.m.
$= \cdot 1174$ c.m.

Change of Density with Temperature.—If M denote the mass of a body in grammes say, V its volume in c.cm., and d its density in grms. per c.cm., then clearly $M = Vd$.

Now when a body is heated it, in general, expands, and since its mass remains constant, its density evidently decreases. If V_o be its volume at 0° C., V_t its volume at t° C., then—

$$M = V_o d_o \qquad M = V_t d_t$$

$$\therefore\ V_t d_t = V_o d_o,$$

i.e.
$$\frac{d_t}{d_o} = \frac{V_o}{V_t} = \frac{V_o}{V_o(1+ct)} = \frac{1}{1+ct}$$

$$\therefore\ d_t = \frac{d_o}{1+ct} \quad \dots\dots\dots\dots\dots\dots\dots\dots (10)$$

This is true, as it stands, for solids, liquids, and gases ; but for solids and some liquids we may have, when t is small enough, an approximate formula, giving—

$$d_t = \frac{d_o}{1+ct} = d_o \frac{1}{1+ct} = d_o(1-ct) \dots\dots\dots (11)$$

Also corresponding to formula (3), above, we have :—

$$\frac{d_t}{d_{t_1}} = \frac{V_{t_1}}{V_t} = \frac{V_o(1+ct_1)}{V_o(1+ct)} = \frac{1+ct_1}{1+ct} = [1 + c(t_1 - t)].$$

$$\therefore\ d_{t_1} = \frac{d_t}{[1 + c(t_1 - t)]} \quad \dots\dots\dots\dots\dots (12)$$

Worked Examples.—(8) *The density of a piece of glass at* 10° *C. is* 2·6, *and at* 60° *C. it is* 2·5966. *Find the mean coefficient of cubical expansion of glass between* 10° *C. and* 60° *C.*

$$\frac{d_t}{d_{t_1}} = [1 + c(t_1 - t)],$$

$$\frac{2{\cdot}6}{2{\cdot}5966} = 1 + 50c, \quad \therefore\ 1{\cdot}00131 = 1 + 50c,$$

i.e.
$$c = \frac{{\cdot}00131}{50} = {\cdot}000026.$$

(9) *The density of mercury at* 0° *C. is* 13·596. *Find its density at* 100° *C., the mean coefficient of cubical expansion of mercury between* 0° *C. and* 100° *C. being* ·000181.

From—
$$d_t = \frac{d_o}{1+ct},$$

we have :—
$$d_{100} = \frac{13{\cdot}596}{1{\cdot}0181} = 13{\cdot}354.$$

(10) *Find the weight of a cubic centimetre of mercury at* 10° *C., having given that, one cubic centimetre of mercury weighs, at* 0° *C.,* 13·596 *grams, and that its mean coefficient of expansion is* ·000181.

Weight is proportional to mass, and, since $M = Vd$, mass is proportional to density when the volume is constant. Hence, if w_t and w_o denote the weights of a given volume at t° C. and 0° C., we have—

$$\frac{w_t}{w_o} = \frac{d_t}{d_o} = \frac{1}{1+ct}.$$

$$\therefore \frac{w_t}{w_o} = 1 - ct, \quad \text{or} \quad w_t = w_o(1 - ct).$$

Hence, $w_t = 13{\cdot}596\,[1 - (10 \times {\cdot}000181)]$
$= 13{\cdot}596 - 13{\cdot}596\,({\cdot}00181)$
$= 13{\cdot}571$ grams.

Exercises II.

✓(1) A brass and a steel rod are one metre long at 10° C.; find the difference in their lengths at 60° C.

(2) A platinum wire is found to be 0·013 cm. longer at 60° C. than at 40° C. Find the length of the wire at 0° C.

✓(3) The volume of a mass of lead at 50° C. is 50 c.cm., and at 80° C. its volume is found to be 50·126 c.cm. Show that the mean coefficient of cubical expansion of lead between 50° C. and 80° C. is approximately 0·000084.

✓(4) A silver rod, one inch in diameter at 0° C., just fits into a copper tube at 100° C. At what temperature will it fit into a glass tube of the same diameter as the copper tube at 0° C?

✓(5) A rod of zinc is 60 cm. long at 10° C. At what temperature will it be 60·2 cm. long?

✓(6) A steel scale gives correct measurements at 62° F. What length will a rod 1 metre long at 10° C. appear to be when measured with this scale?

✓(7) A glass rod of volume V at 0° C. exactly fills a hollow brass cylinder at 10° C. Find the volume of the space between the rod and the cylinder at 50° C.

(8) Two metal rods have the same length, one metre, at 100° C.; but at 80° C. the difference between their lengths is 0·02 cm. If the mean coefficient of linear expansion of the less expansible rod be ·0·000015, find the corresponding coefficient for the other rod, and the length of each rod at 0° C.

(9) Find the mean coefficient of linear expansion of iron on the Fahrenheit scale, the initial length being referred to zero on the same scale.

(10) The volume of a given mass of lead at $-30°$ C. is represented by V. If c denote the mean coefficient of cubical contraction between 0° C. and $-30°$ C., and c' the mean coefficient of cubical expansion between 0° C. and 100° C., find the volume of the lead at 100° C., and express the mean coefficient of cubical expansion of lead between $-30°$ C. and 100° C. in terms of c and c'.

(11) Compare the density of lead at 100° C. with its density at $-100°$ C., assuming its coefficient of expansion to remain constant within these limits of temperature.

(12) The mean coefficient of expansion of a substance between $-t°$ C. and 0° C. is c, and the mean coefficient of expansion between 0° C. and T° C. is c'. Compare the density of the substance at each of these temperatures with its density at 0° C.

(13) Find the mass of a cubic centimetre of silver at 250° C., the density of silver at 0° C. being 10·31 grams per c.c.

(14) The density of water at 0° C. is 0·999871, and at 4° C. it is 1. Find the mean coefficient of expansion of water between 4° C. and 0° C.

(15) If δ denote the density of a substance at $t°$ C., and δ' its density at $t'°$ C., find the mean coefficient of expansion of the substance between $t°$ C. and $t'°$ C.

(16) Find the weight, in grams, of 10 c.cm. of gold (density 19·4 grams per c.cm. at 0° C.) at 15° C.

(17) The density at 0° C. of a specimen of wrought iron is 7·3, and the density at 0° C. of a specimen of tin is 7·4: at what temperature will these two specimens have the same density?

(18) The steel rails of a railway are 8 yards long at 0° C., and they are laid at a temperature of 15° C. What spaces must be left between them to allow of a variation in temperature ranging from 20° F. to 90° F.?

(19) A glass ball at a temperature of 40° C. is dropped into a vessel full of water also at 40° C., and it is observed that a pint of water overflows. Find what volume would overflow if the same glass ball at 100° C. was dropped into the same vessel at 100° C., being given that the coefficient of cubical expansion of glass is ·000025.

(20) The volume of a certain mass of ice at 0° C. is 105 cub. cm., and it is observed that when loaded with a mass of 10 grammes it just floats in water at 0° C., totally immersed but with the 10 grammes above the surface. Determine from this the change of volume of 1 cub. cm. of water on freezing.

CHAPTER III.

EXPANSION OF LIQUIDS.

Formulae for Calculations.—(1) *Apparent and Real Expansions.* The mean coefficient of real expansion (c_r) of a liquid is **approximately** equal to the sum of the mean coefficient of apparent expansion (c_a) of the liquid in any vessel and the mean coefficient of volume expansion (c) of that vessel, *i.e.*

$$c_r = c_a + c \qquad (1)$$

The relation between the *apparent* volume V_a at $t°C$ and the *true* volume V_o at 0°C. is

$$V_a = V_o(1 + c_a t) \qquad (2)$$

The relation between the *true* volume V_t at $t°C$ and the *apparent* volume V_a at $t°C$ is

$$V_t = V_a(1 + ct) \qquad (3)$$

(2) *Application of above to Linear Expansion.*—The above applies directly to volume expansion but it also applies to linear expansion. Thus if we are measuring the length of a brass rod with a steel scale and if l_r be the mean coefficient of real linear expansion of the brass rod, l_a its mean coefficient of apparent expansion, and l the mean coefficient of linear expansion of the steel scale,

$$l_r = l_a + l \qquad (4)$$

corresponding to (1) above. Similarly corresponding to (2) and (3) above we have:—

$$L_a = L_o(1 + l_a t) \qquad (5)$$

$$L_t = L_a(1 + lt) \qquad (6)$$

where the letters have their usual meaning.

Weight Thermometer.—The formulae for calculations on the weight thermometer are :—

$$c_a = \frac{w'}{(W_o - w')T} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where W_o is the weight of liquid in the thermometer at 0° C. and w' the weight of liquid which overflows when the temperature is raised from 0° C. to T° C. c_a = coefficient of apparent expansion of the liquid. From the above :—

$$T = \frac{w'}{(W_o - w')c_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

The real coefficient c_r for the liquid may be found approximately from the relation $c_r = c_a + c$, but the exact relation is :—

$$c_r = c_a + \frac{W_o}{W_o - w'}\, c,$$

c being the volume coefficient for the vessel.

Hydrostatic Method of finding the Cubical Expansion of a Solid or Liquid.—The formulae for calculations are :—

$$w_t(1 + ct) = w_o(1 + gt) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

where w_o = *loss* of weight of solid in liquid at 0° C., w_t = *loss* of weight of solid in liquid at t° C., c = mean coefficient of expansion of the liquid between 0° C. and t° C., and g = mean coefficient of expansion of the solid between 0° C. and t° C. Thus if c be known, g is found, or if g be known, c is found.

For calculations, this formula may be more conveniently expressed as an approximation, thus :—

$$\frac{w_o}{w_t} = \frac{1 + ct}{1 + gt}.$$

$$\therefore \frac{w_o}{w_t} = 1 + (c - g)t. \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

and for two temperatures t and t_1 (instead of t° and 0°) we have the approximate relation :—

$$\frac{w_{t1}}{w_t} = 1 + (c - g)(t - t_1) \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

Worked Examples.—(1) *A zinc rod is measured by means of a brass scale, and found to be* 1·0001 *metres long at* 10° *C. What is the real length of the rod at* 0° *C. and at* 10° *C.?* [*Mean coefficient of linear expansion of zinc is* ·000029 *and of brass* ·000019.]

Applying (5) above we get—

$$L_a = L_o(1 + l_a t).$$

$$\therefore\ L_o = \frac{L_a}{1 + l_a t} = L_a(1 - l_a t).$$

Here—

$$L_a = 1{\cdot}0001;\ l_a = (l_r - l) = ({\cdot}000029 - {\cdot}000019)$$
$$= {\cdot}00001;\ \text{and } t = 10.$$
$$\therefore\ L_o = 1{\cdot}0001[1 - ({\cdot}00001)10]$$
$$= 1{\cdot}0001[1 - {\cdot}0001]$$
$$= 1 \text{ nearly}.$$

Also— $$L_t = L_o(1 + l_r t).$$

Here— $$L_o = 1 \text{ nearly, and } l_r = 0{\cdot}000029.$$
$$\therefore\ L_{10} = 1\,(1 + 0{\cdot}00029)$$
$$= 1{\cdot}00029.$$

Or— $$L_t = L_a(1 + lt).$$
$$\therefore\ L_{10} = 1\,0001\,(1 + {\cdot}000019 \times 10)$$
$$= 1{\cdot}0001\,(1 + {\cdot}00019)$$
$$= 1{\cdot}00029.$$

(2) *A glass tube is measured with a steel scale, correct at* 0° *C., and found to be* 70 *cm. long at* 0° *C., and* 69·99811 *cm. long at* 10° *C. Find the mean coefficient of real expansion of glass, and also its mean coefficient of apparent expansion with reference to steel.* [*The mean coefficient of expansion of steel is* 0·0000113.]

$$L_o = L_a[1 - l_a t]$$
$$70 = 69{\cdot}99811[1 - 10l_a]$$
$$70 = 69{\cdot}99811 - 699{\cdot}9811 l_a$$
$$699{\cdot}9811 l_a = 69{\cdot}99811 - 70$$
$$699{\cdot}9811 l_a = -0{\cdot}00189.$$
$$\therefore\ l_a = -\frac{{\cdot}00189}{699{\cdot}9811} = -0{\cdot}0000027.$$

That is, the mean coefficient of apparent expansion of glass relative to steel is negative. This explains why the tube is *apparently* shorter at 10° C. than at 0° C.

Also—
$$l_r = l + l_a, \text{ and } l = 0{\cdot}0000113.$$
$$\therefore\ l_r = 0{\cdot}0000113 - 0{\cdot}0000027$$
$$= 0{\cdot}0000086.$$

(3) *An ordinary mercurial thermometer is placed with its bulb and lower part of stem in a vessel of water, and indicates a temperature T°. The upper portion of the stem, containing n divisions of the mercury column, is in the air at a temperature t°. What is the true temperature of the water in the vessel?*

The true temperature of the water, T′°, is that which the thermometer would indicate if completely immersed in the water. If this were the case, the n divisions of the mercury column, which are now at $t°$, would be at T′°. We have therefore to find the expansion of these n divisons for a rise in temperature from $t°$ to T′°. If c_a denote the mean coefficient of apparent expansion of mercury in glass we have—

$$n' = n[1 + c_a(\mathrm{T}' - t)]$$

But—
$$\mathrm{T}' = (\mathrm{T} - n) + n'$$
$$= \mathrm{T} + n' - n$$
$$= \mathrm{T} + n(\mathrm{T}' - t)c_a.$$
$$\therefore\ \mathrm{T}' = \frac{\mathrm{T} - ntc_a}{1 - nc_a} \quad \text{.........................} (12)$$

(4) *A weight thermometer weighs* 50 *grams when empty, and* 710 *grams when full of mercury at* 0° *C. On heating up to* 100° *C.,* 10 *grams of mercury are expelled. Calculate the mean coefficient of cubical expansion of glass, assuming the mean coefficient of real cubical expansion of mercury to be* 0·000181.

Applying (7) above we get—

$$c_a = \frac{w'}{(\mathrm{W}_o - w')\mathrm{T}}.$$
$$\therefore\ c_a = \frac{10}{(660 - 10)100} = \frac{1}{6500}.$$
$$\therefore\ c_a = 0{\cdot}000154 \text{ nearly}.$$

But— $c_r = c_a + c.$

$$\therefore\ c = c_r - c_a$$
$$= 0\cdot000181 - 0\cdot000154$$
$$= 0\cdot000027.$$

(5) *The same weight thermometer is employed to determine the expansion of glycerine. Its weight, full of glycerine at* 0° C. = 163·13 *grams, and at* 100° C. = 157·65 *grams. Find the mean coefficient of real cubical expansion of glycerine.*

As above— $c_a = \dfrac{w'}{(W_o - w')T}.$

Here $w' = 5\cdot48$, $W_o = 163\cdot13 - 50 = 113\cdot13$, and $T = 100$.

$$\therefore\ c_a = \frac{5\cdot48}{107\cdot65 \times 100} = 0\cdot00051 \text{ nearly} \quad \text{and } c_r = c_a + c$$
$$\therefore\ c_r = 0\cdot00051 + 0\cdot000027 = 0\cdot000537.$$

(6) *The weight thermometer of questions* 4 *and* 5, *is employed to determine the temperature of an oil bath. After it has taken the temperature of the bath, it is found that* 30 *grams of mercury have been expelled. Find the temperature of the bath.*

From (8) we have—

$$T = \frac{w'}{(W_o - w')c_a}. \qquad \text{Let } c_a = \frac{1}{6500}$$
$$T = \frac{30}{(660 - 30)\frac{1}{6500}} = \frac{6500 \times 30}{630}$$
$$\therefore\ T = 309\cdot5^\circ \text{ C. nearly.}$$

(7) *A glass bulb, with a uniform fine stem, weighs* 10 *grams when empty,* 117·3 *grams when the bulb only is full of mercury, and* 119·7 *grams when a length of* 10·4 *cm. of the stem is also filled with mercury; calculate the relative coefficient of expansion for temperature of a liquid which, when placed in the same bulb, expands through the length from* 10·4 *to* 12·9 *of the stem, when warmed from* 0° *C. to* 28° *C. The density of mercury is* 13·6 *grams per c.cm.*

From the question, 10·4 cm. of the bore of the stem con-

tain $119 \cdot 7 - 117 \cdot 3 = 2 \cdot 4$ grams of mercury. $\therefore$ 1 cm. of bore of stem contains $\frac{2 \cdot 4}{10 \cdot 4} = \frac{3}{13}$ grams of mercury.

Let v denote the volume of 1 cm. of bore of stem, then volume of bulb is equivalent to that of $\frac{117 \cdot 3 - 10}{\frac{3}{13}} v = \frac{107 \cdot 3 \times 13}{3} v$ cm. of stem $= 464 \cdot 97v$ nearly.

$\therefore$ Initial volume of liquid $= (464 \cdot 97 + 10 \cdot 4)v = 475 \cdot 37v$ nearly, and final volume $= (464 \cdot 97 + 12 \cdot 9)v = 477 \cdot 87v$.

Now in the case of a liquid being heated in a graduated glass tube, if n be the reading on the stem at 0° C. and n' the reading at t° C., it follows that—

$$c_a = \frac{n' - n}{nT}$$

and applying this, we get:—

$$c_a = \frac{2 \cdot 5v}{28 \times 475 \cdot 37v} = \frac{2 \cdot 5}{475 \cdot 37 \times 28} = 0 \cdot 000188 \text{ nearly.}$$

The operations with mercury, referred to in the first part of the question, constitute a simple method of calibration.

A more usual way of working such a question is to deduce the volume of the bulb, etc., in c.cm., by employing the known density of mercury (13·6), thus:—

$$\text{Volume of 1 cm. of stem} = \frac{2 \cdot 4}{10 \cdot 4 \times 13 \cdot 6} \text{ c.cm.} = \frac{3}{13 \times 13 \cdot 6} \text{ c.cm.}$$

$$\text{Total apparent expansion} = \frac{2 \cdot 5 \times 3}{13 \times 13 \cdot 6}.$$

$\therefore$ Mean coefficient of apparent expansion between 0° C. and 28° C.

$$= \left(\frac{2 \cdot 5 \times 3}{13 \times 13 \cdot 6 \times 28} \right) \div \frac{109 \cdot 7}{13 \cdot 6}$$

$$= \frac{2 \cdot 5 \times 3 \times 13 \cdot 6}{13 \times 13 \cdot 6 \times 28 \times 109 \cdot 7} = \frac{2 \cdot 5 \times 3}{13 \times 28 \times 109 \cdot 7} = 0 \cdot 000188 \text{ nearly.}$$

(8) *A piece of glass weighs 47 grams in air, 31·53 grams in water at 4° C. and 31·75 grams in water at 60° C. Find the mean coefficient of cubical expansion of water between 4° C. and 60° C., taking that of glass as 0·000024.*

From (11) we have—

$$\frac{w_4}{w_{60}} = 1 + (c - g)(60 - 4).$$

Here— $w_{60} = 47 - 31{\cdot}75 = 15{\cdot}25$ grams,

$w_4 = 47 - 31{\cdot}53 = 15{\cdot}47$ grams,

$g = 0{\cdot}000024.$

Hence— $$\frac{15{\cdot}47}{15{\cdot}25} = 1 + (c - g)\,56,$$

$$1{\cdot}014426 = 1 + (c - g)\,56.$$

$$\therefore\ c - g = \frac{0{\cdot}014426}{56} = 0{\cdot}000257.$$

That is— $$c = 0{\cdot}000257 + g.$$

$$\therefore\ c = 0{\cdot}000257 + 0{\cdot}000024 = 0{\cdot}000281.$$

A simple example such as this may be, very instructively, worked out from first principles. The loss of weight of the glass in water at 4° C. = 15·47 grams; ∴ its volume = 15·47 c.cm., for a gram is, by definition, the weight of 1 c.cm. of water at 4° C. Hence, volume of glass at 60° = 15·47[1 + (0·000024 × 56)] = 15·4908 c.cm.

Hence, when weighed in water at 60° C., it displaces this volume of water, and therefore we have, that 15·4908 c.cm. of water at 60° weigh 15·25 grams. This weight of water would occupy 15·25 c.cm. at 4° C., and hence we have—

$$15{\cdot}4908 = 15{\cdot}25\,(1 + 56c).$$

$$\therefore\ c = \frac{0{\cdot}2408}{15{\cdot}25 \times 56} = 0{\cdot}000282.$$

This result agrees very closely with the approximation obtained above.

(9) *A solid weighs* 29·9 *grams in a liquid of density* 1·21 *at* 10° *C. It weighs* 30·4 *grams in the same liquid at* 95° *C. when the density of the liquid is* 1·17. *Calculate the coefficient of cubical expansion of the solid, given that its weight in air* = 45·6 *grams.*

From (11) we have—

$$\frac{w_{t_1}}{w_t} = 1 + (c - g)(t - t_1).$$

Here— $w_{t_1} = w_{10} = 45{\cdot}6 - 29{\cdot}9 = 15{\cdot}7$

$w_t = w_{95} = 45{\cdot}6 - 30{\cdot}4 = 15{\cdot}2.$

And therefore— $\dfrac{15\cdot7}{15\cdot2} = 1 + (c - g)(95 - 10),$

or — $1\cdot0329 = 1 + 85(c - g).$

$$\therefore\ c - g = \frac{\cdot0329}{85} = \cdot000387.$$

Here g is required, and c has to be calculated from the densities of the liquid at 10° C. and 95° C. Now :—

$$\frac{d_{10}}{d_{95}} = [1 + (95 - 10)c] = 1 + 85c.$$

That is — $\dfrac{1\cdot21}{1\cdot17} = 1 + 85c,$

or— $1\cdot034188 = 1 + 85c.$

$$\therefore\ c = \frac{0\cdot034188}{85} = 0\cdot000402.$$

Hence for g we have—

$$0\cdot000402 - g = 0\cdot000387.$$
$$\therefore\ g = 0\cdot000015.$$

Barometric Correction for Temperature.—The pressure of the atmosphere is usually expressed in terms of the height of a column of mercury at 0° C. which exerts an equivalent pressure. The *observed* height of a barometer at t° C. has to be reduced to the *equivalent* height at 0° C.; this is called correcting for temperature, and in applying the correction two points are to be remembered. (i) To correct for the expansion of the scale between 0° C. and t° C. (ii) To correct for the change of density of the mercury.

Let H denote the *observed* height of the barometer at t° C. Then if l denote the mean coefficient of linear expansion of the scale, the true height, corrected for the expansion of the scale, is $H(1 + lt)$. Denote this by H_t.

To correct for (ii): If H_t and d_t be the height and density of the mercury at t, H_0 the equivalent height at 0° C. and d_0 the density at 0° C., then since they are equivalent and denote equal pressures per unit area we have :—

$$H_t d_t g = H_0 d_0 g$$

where g = acceleration due to gravity. Hence

$$H_t d_t = H_0 d_0.$$

That is— $$H(1 + lt)d_t = H_o d_o.$$

$$\therefore\ H_o = H(1 + lt)\frac{d_t}{d_o}.$$

But— $$\frac{d_t}{d_o} = \frac{1}{1 + c_r t}.$$

$$\therefore\ H_o = H\frac{1 + lt}{1 + c_r t} = H[1 + (l - c_r)t].$$

Or, since c_r is usually greater than l, this is more generally written $$H_o = H[1 - (c_r - l)t]$$

where H denotes the observed height at $t°$ C.

H_o „ „ true „ at 0° C.

l „ „ mean coefficient of expansion of the scale.

c_r „ „ „ „ absolute expansion of mercury.

t „ „ temperature of observation.

It should be noticed that the mean coefficient of *absolute cubical* expansion of mercury is employed because the correction (ii) is necessary on account of change of density of the mercury. On no account must the column of mercury be treated as a rod subject to linear expansion.

Sometimes a barometer reading must be reduced to 0° C., to sea level and to latitude 45°. In this connection it may be mentioned that if θ = latitude and h = height above sea level *in feet*, the height of the standard barometer is :—

$$\overline{H} = 760 + 1{\cdot}94562 \cos 2\theta + {\cdot}000045466\, h \text{ mm.}$$

This will enable the student to take latitude and altitude into account if necessary (see worked example (11)).

Worked Examples.—(10) *A barometer with a glass scale reads* 755 *mm. at* 18° *C.; find the reading at* 0° *C. The apparent coefficient of expansion of mercury in glass is* ·000155, *and the mean coefficient of linear expansion of glass is* ·0000089.

Applying the above formula, we have $H_o = H[1-(c_r - l)t]$.

To find c_r from c_a which is given, we have $c_r = c_a + c$.

Here $c_a = \cdot000155$ and $c = 3(\cdot0000089) = \cdot0000267$.

$$\therefore\ c_r = \cdot000155 + \cdot0000267 = \cdot0001817.$$

Then— $H_o = 755[1 - (\cdot0001817 - \cdot0000089)18]$

$$= 755[1 - (\cdot0001728)\,18]$$
$$= 755[1 - \cdot00311]$$
$$= 752\cdot7 \text{ nearly.}$$

(11) *A barometer in Leeds (lat. 53° 48·7′) was 185·8 feet above sea level; find the length of the column which corresponds to a standard atmosphere. When the barometer, whose scale was made of brass, read 765·27 mm. at a temperature of 12·5° C., a thermometer immersed in a boiling-point apparatus read 100·37°. Find the correction. (Take mercury coefficient as ·000182 and brass coefficient as ·000019.)*

Using the formula above we have:—

Height required $= 760\cdot000 + 1\cdot94562 \cos 107°\ 37\cdot4'$
$+ \cdot000045466 \times 185\cdot8$ mm.
$= 760\cdot000 - \cdot588 + \cdot008 = 759\cdot42$ mm.

Hence the standard atmospheric pressure at Leeds is 759·42 mm.

It therefore follows that for readings of the barometer at Leeds near 760 mm., it is sufficiently accurate to add ·58 mm. to the reading of the barometer after it has been corrected for temperature.

The reading of the barometer therefore, corrected for temperature and for latitude and height above sea-level

$$= 765\cdot27\,[1 - (\cdot000182 - \cdot000019)12\cdot5] + \cdot58 \text{ mm.}$$
$$= (763\cdot72 + \cdot58 \text{ mm.}) = 764\cdot30 \text{ mm.}$$

From Regnault's Tables, boiling point at this pressure
$= 100\cdot16°$ C.

Observed boiling point $= 100\cdot37°$ C.

$\therefore$ Correction $= -\cdot21°$ C.

Exercises III.

(1) A rod of copper and a rod of iron placed side by side are riveted together at one end. The iron rod is 150 cm. long, and a mark is made on the copper rod, showing the position of the unriveted end of the iron at 0° C. If at 30° the mark is 0·0255 cm. from the end of the iron rod, what is the coefficient of expansion of copper, that of iron being 0·000012?

(2) A weight thermometer weighed 95·6 grams when empty, and 676·8 grams when full of mercury at 0° C.; find the quantity of mercury expelled when the temperature is raised to 100° C.

(3) The coefficient of absolute cubical expansion of mercury is ·00018, the coefficient of linear expansion of glass is ·000008. Mercury is placed in a graduated glass tube, and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101·56 divisions?

(4) A porcelain weight thermometer weighs 165 grams when empty and 468 grams when full of mercury at 0° C. When heated to 300° C., the weight of overflow is found to be 13·464 grams. Find the mean coefficient of cubical expansion of porcelain between 0° and 300° C., assuming that of mercury to be ·000184 for the same range of temperature.

(5) A solid weighs 320 grams *in vacuo.*, 240 grams in distilled water at 4° C., and 242 grams in water at 100° C., of which the density is 0·959. Find the volume of the solid at these two temperatures, and deduce therefrom its mean coefficient of cubical expansion for 1° C.

(6) The barometer height at 12° C., as indicated by a barometer with a brass scale, is 766·45 cm. Find the true equivalent height at 0° C.

(7) A glass bulb with a straight graduated capillary stem weighs 54 grams when empty, 367 grams when filled with mercury, at 0° C., up to the zero of the graduations on the stem, and 367·12 grams when filled with mercury, at 0° C., up to the division marked 100 on the stem. Find the mean coefficient of absolute cubical expansion of a liquid which fills the bulb and stem up to the tenth division at 0° C. and up to the 110th division at 100° C.

(8) What will be the reading of a mercury thermometer if the bulb is at 300° C., and the stem from 0° C. upwards at 20° C.? The coefficient of expansion for mercury may be taken as ·000187 and the linear coefficient of glass ·000009.

(9) A glass flask holds 100 c.cm. of mercury at 0° C. Find the volume at 100° C. of the mercury driven out when the flask and mercury are heated to 100° C. (coefficient of cubical expansion of glass = ·000026 : of mercury ·00018)

(10) The reading of a mercury barometer is 760 mm. when the temperature is 0° C. Determine the reading at 20° C., the coefficients of expansion of mercury and of the metal of which the scale is made being respectively $1·8 \times 10^{-4}$ and $1·8 \times 10^{-5}$.

(11) A metre scale, linear coefficient = ·000011, is tested at 0° C., and found to be shorter than the standard by half a millimetre. What error in metres would be involved in the measurement of a distance of 200 kilometres by this scale at 20° C. ?

(12) A piece of metal has an apparent weight of 29·9 grams in brine of density 1·21 grams per cub. cm. at 10° C., but has an apparent weight of 30·4 grams in the same liquid at 95° C., at which temperature the density of the liquid is 1·17 grams per cub. cm. If the weight in air of the piece of metal is 45·6 grams, calculate its coefficient of cubical expansion.

(13) A barometer with a brass scale which has been adjusted at 0° C. stands at 778 mm. when the temperature is 20° C. What pressure in kilograms per sq. cm. does this indicate ?

(Coefficient of linear expansion of brass = 0·0000188 ;
,, cubical ,, ,, mercury = 0·0001803 ;
Weight of one cubic centimetre of mercury at 0° C. = 13·596.)

(14) Assuming that the mean coefficient of expansion of mercury for 1° C. is ·0001815, and that of the glass of a thermometer ·000026, find the reading of such a thermometer, of which the bulb is plunged in water at the temperature of 100° C., while the stem is exposed to air of the temperature of 10° C.

(15) Suppose that an English barometer with a brass scale giving true inches at the temperature 62° F. reads 29·5 inches at 45° F ; what is the pressure in true inches of mercury reduced to the specific gravity it has at 32° F. ? [The coefficient of linear expansion of brass is 0·00001, and that of the cubical expansion of mercury is 0·0001 ; both for 1° F.]

(16) A weight thermometer was used as a maximum thermometer. When empty the thermometer weighed 4·98 grams. When filled with a given liquid at the temperature of melting ice it weighed 150·43 grams. It was now exposed in the room whose maximum temperature during a certain time was required. At the end of that time it was removed and weighed : the weight was only 147·42 grams. The real coefficient of the given liquid is ·000237 and the coefficient of volume expansion of glass is ·000026. Find the maximum temperature of the room during the given period.

CHAPTER IV.

EXPANSION OF GASES.

Formulae for Calculations.—The formulated expressions of the gaseous laws are:—

Boyle's Law (*temperature constant*) $pv = k$ .... (1)

Charles' Law (*pressure constant*) $v_t = v_o (1 + c_v t)$ ...... (2)

Law of Pressures (*volume constant*) $p_t = p_o (1 + c_p t)$... (3)

where c_v = coefficient of expansion of the gas at constant pressure, and c_p = the pressure coefficient at constant volume. k is a constant for the given mass of the given gas; it is, of course, different for different masses of the gas.

We are dealing with Centigrade temperatures and experiment shows that in this case $c_v = c_p = \frac{1}{273}$ (or more exactly 1/273·7). Using the Fahrenheit scale, the coefficients are $\frac{5}{9}$ of 1/273·7, *i.e.* $\frac{1}{493}$, and in this case (2), for example, would be written:—

$$v_t = v_{32} \{1 + \tfrac{1}{493} (t - 32)\}.$$

When **the temperature, volume and pressure are all changing**, the laws become:—

$$\frac{pv}{T} = \text{constant} = R. \quad \therefore pv = RT \text{ .... (4)}$$

or, expressed in a convenient form for calculation purposes:—

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \text{ .... (5)}$$

where T, T_1 and T_2 denote **absolute temperatures.** The absolute zero is $-273°$ C. (approx.) so that any temperature $t°$ C. is $(273 + t)°$ absolute.

Another form of the relation which is sometimes convenient for calculations is :—

$$\frac{p_1 v_1}{1 + c_r t_1} = \frac{p_2 v_2}{1 + c_r t_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where the letters have their usual meaning. (See Example 7.)

In (5) *if the temperature be constant*, we get $p_1 v_1 = p_2 v_2$, which is another expression for (1) above.

In (5) *if the pressure be constant*, we get $v_1/T_1 = v_2/T_2$, that is :—

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

or in words, "the volume is proportional to the absolute temperature" which is another form of (2) above.

Finally in (5) *if the volume be constant*, we get $p_1/T_1 = p_2/T_2$, that is :—

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

which is another form of (3) above.

Note that the absolute zero is also $-461°$ F. so that the F. freezing point is $(461 + 32) = 493°$ absolute and, in fact, any temperature $t°$ F. is $(461 + t)°$ absolute.

Mass of a Given Volume of Dry Air.—This is often required and is obtained as follows :—

1 c.cm. of dry air at 0° C. (273° abs.) and 760 mm. pressure

$$= \cdot 001293 \text{ grms.} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

∴ V c.cm. of dry air at $t°$ C. $\{(273 + t)° \text{ abs.}\}$ and P mm. pressure

$$= V \times \cdot 001293 \times \frac{273}{273 + t} \times \frac{P}{760} \text{ grms.} \ldots\ldots \quad (10)$$

See Example (9), p. 30.

Work done by a Gas Expanding at Constant Pressure.—Suppose the gas in a cylinder fitted with a piston. Let

P = pressure per unit area, A = area of piston, l = distance through which the piston moves. Then:—

$$W = \text{Work} = P \times A \times l = P.v \text{............} (11)$$

where v *is the increase in volume.* With regard to units:—

If P is in:—	and v in:—	W will be in:—
dynes per sq. cm.	c.cm.	ergs
grams per sq. cm.	c.cm.	centimetre-grams
pounds per sq. ft.	c.ft.	foot-pounds

Again let P = pressure, V = volume and T = absolute temperature of a given mass of gas. Let the temperature **be raised one degree** at constant pressure. The *increase in volume* (v) will be $\frac{V}{T}\left\{ i.e. \frac{1}{T} \text{ of vol. at T (from (7))} \right\}$; hence:—

$$W = \text{Work} = Pv' = \frac{PV}{T} \text{ .... } (12)$$

See Example (10) p. 31.

Worked Examples.—(1) *A cubic metre of gas at 760 mm. pressure is subjected, at constant temperature, to a pressure of 2,280 mm. Find its volume.*

Here, in— $p_1v_1 = p_2v_2$

we have— $p_1 = 760$ mm.; $p_2 = 2{,}280$ mm.

$v_1 = 1$ cubic metre; v_2 is required.

Hence $760 \times 1 = 2280\, v_2$

Or— $v_2 = \frac{760}{2280} = \frac{1}{3}$ cubic metre.

(2) *In a vessel of capacity* 5 *litres are placed* 15 *litres of hydrogen originally at* 120 *mm. pressure and* 10 *litres of oxygen originally at* 500 *mm. pressure. Find the pressure of the mixed gases (temperature constant).*

Hydrogen :—

$$P_2V_2 = P_1V_1. \quad \therefore P_2 = \frac{P_1V_1}{V_2} = \frac{120 \times 15}{5} = 360 \text{ mm.}$$

Oxygen :—

$$P_2V_2 = P_1V_1. \quad \therefore P_2 = \frac{500 \times 10}{5} = 1000 \text{ mm.}$$

$$\therefore \text{Total pressure} = 360 + 1000 = 1360 \text{ mm. of mercury.}$$

[Note that each gas is regarded as "filling" the vessel, *i.e.*, V_2 for each gas is 5 litres.]

(3) *A litre of hydrogen, at* 10° *C., is heated at constant pressure to* 293° *C. Find its volume.*

Here, in—

$$\frac{v_1}{v_2} = \frac{T_1}{T_2}$$

we have—

$$v_1 = 1 \text{ litre}; \; T_1 = 273 + 10 = 283,$$

$$T_2 = 273 + 293 = 566; \; v_2 \text{ is required.}$$

Hence—

$$\frac{1}{v_2} = \frac{283}{566} = \frac{1}{2},$$

$$\therefore v_2 = 2 \text{ litres.}$$

(4) *Air is enclosed in a vessel at* 0° *C., and, the volume being kept constant, the temperature is lowered to* — 88° *C., at which temperature the pressure is found to be* 385 *mm. Find the pressure at* 0° *C.*

Here in—

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

we have— p_1 is required; $T_1 = 273$.

$$p_2 = 385 \text{ mm.}; \; T_2 = 273 + (-88) = 273 - 88 = 185.$$

Hence—

$$\frac{p_1}{385} = \frac{273}{185},$$

$$\therefore p_1 = 568 \cdot 1 \text{ mm.}$$

(5) *A gas occupies* 5·5 *c. ft. at* 32° *F. Find its volume at* 212° *F., the pressure being constant.*

$v_1 = 5{\cdot}5$ c. ft.; $T_1 = 461 + 32 = 493$; $T_2 = 461 + 212 = 673$; v_2 is required.

$$\frac{v_2}{v_1} = \frac{T_2}{T_1}, \quad i.e. \quad \frac{v_2}{5{\cdot}5} = \frac{673}{493} \quad \therefore\ v_2 = \frac{673 \times 5{\cdot}5}{493},$$

$$i.e.\ v_2 = 7{\cdot}5 \text{ c. ft.}$$

(6) *Find the volume occupied at* 0° *C. and* 760 *mm. pressure by* 500 *c.c. of oxygen measured at* 10° *C. and* 750 *mm. pressure.*

Here, in—
$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$
we have—
$$p_1 = 750 \text{ mm.};\ p_2 = 760 \text{ mm.}$$
$$v_1 = 500 \text{ c.c.};\ v_2 \text{ is required.}$$
$$T_1 = 273 + 10 = 283;\ T_2 = 273 + 0 = 273.$$

Hence—
$$\frac{750 \times 500}{283} = \frac{760 v_2}{273}.$$

Or—
$$v_2 = \frac{273 \times 750 \times 500}{283 \times 760}$$
$$= 480{\cdot}6 \text{ c.c.}$$

(7) *A thousand cubic centimetres of air at* 50° *C. are cooled down to* 10° *C., and at the same time the external pressure upon the air is increased from* 750 *mm. to* 765 *mm. What is the volume reduced to, the coefficient of expansion of air for* 1° *C. being* 0·00366?

Using the relation:—
$$\frac{p_1 v_1}{1 + c_r t_1} = \frac{p_2 v_2}{1 + c_r t_2},$$
we have—

$$p_1 = 750 \text{ mm.} \qquad p_2 = 765 \text{ mm.}$$
$$v_1 = 1000 \text{ c.cm.} \qquad v_2 \text{ is required.}$$
$$t_1 = 50° \text{ C.} \qquad t_2 = 10° \text{ C.}$$
$$c_r = 0{\cdot}00366.$$

$$\therefore \frac{750 \times 1000}{1 + (0{\cdot}00366 \times 50)} = \frac{765 v_2}{1 + (0{\cdot}00366 \times 10)}.$$

$$i.e. \quad \frac{750 \times 1000}{1{\cdot}183} = \frac{765 v_2}{1{\cdot}0366}.$$

$$\therefore\ v_2 = 859{\cdot}06 \text{ c.cm.}$$

(8) *Compare the density of air at 0° C. with that of air at 100° C.*

If we take a given mass of air at 0° C., and heat it to 100° C., the density diminishes; but, since the mass is constant, we have, at any temperature $v_1 d_1 = v_2 d_2$;

$$i.e., \quad \frac{v_1}{v_2} = \frac{d_2}{d_1}.$$

But from (7) we have—

$$\frac{v_1}{v_2} = \frac{T_1}{T_2}. \qquad \therefore \quad \frac{d_2}{d_1} = \frac{T_1}{T_2}.$$

[That is, density is inversely proportional to absolute temperature.]

Here, then— $$\frac{d_0}{d_{100}} = \frac{373}{273} = 1{\cdot}366.$$

(9) *The mass of a litre of air at 0° C. and 760 mm. pressure is 1·293 grams; find the mass of a litre of air at 100° C. and 750 mm. pressure.*

The volume being constant, mass is proportional to density, that is—

$$\frac{M}{M'} = \frac{d}{d'}$$

But— $\dfrac{d}{d'} = \dfrac{T'}{T}$, and, by Boyle's law, $\dfrac{d}{d'} = \dfrac{p}{p'}$.

$$\therefore \frac{M}{M'} = \frac{d}{d'} = \frac{T'}{T} \times \frac{p}{p'}.$$

e. $$M' = M \times \frac{T}{T'} \times \frac{p'}{p}.$$

It will be noticed that this agrees with (10) page 26.

Here, $M = 1{\cdot}293$ grams, $T = 273$, $p = 760$.

M' is required, $T' = 373$, $p' = 750$.

$$\therefore M' = 1{\cdot}293 \times \frac{273}{373} \times \frac{750}{760} = {\cdot}934 \text{ gram.}$$

(10) *Show that the work done by a given mass of gas during expansion under constant pressure, for one degree rise in temperature, is the same for all pressures and temperatures.*

Let V denote the volume of the gas, T its absolute temperature, and P the external constant pressure.

The increase of volume for 1° rise in temperature is evidently $\frac{V}{T}$, and the work done during the expansion is therefore given by $\frac{PV}{T}$. But $\frac{PV}{T}$ is constant for all pressures and temperatures for a given mass of a given gas. Q.E.D.

Exercises IV.

(1) In a determination of the coefficient of expansion of dry air by Gay Lussac's method, the volume of the air in the tube was found to be 240 c.cm. at 0° C., and at 77° C. its apparent volume was 310 c.cm. Find the value obtained for the required coefficient. The mean coefficient of cubical expansion of glass is 0·000026.

✓ (2) A porcelain air thermometer is used to determine the temperature of a furnace. The excess of the pressure of the air in the bulb over the atmospheric pressure is found to be that due to 1843 mm. of mercury. Find the temperature of the furnace, given that the barometric height at the time of determination equals 758 mm.

(3) A volume V of gas at pressure P and temperature T is heated (1) at constant pressure and (2) at constant volume, to a temperature T′. Express in terms of P, V, T, and T′ the resulting *state* in each case. [The *state* of a given mass of gas is expressed by giving its pressure, volume, and temperature.]

✓ (4) Ten litres of hydrogen at 20° C. and 750 mm. pressure, occupy a volume of 9532·4 c.cm. at 10° C. and 760 mm. pressure. Find the mean coefficient of expansion of hydrogen.

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(5) A straight vertical tube, the section of whose bore is one square inch, is closed at its lower end and contains a quantity of air, which supports an air-tight piston whose weight is 1 lb. The position of the piston is observed when the temperature of the air is 31° C., and the weight of the piston is then increased by 1 lb. Find what increase of temperature will be required to bring back the piston to its former position, the atmospheric pressure being 15 lb. per square inch, and the absolute zero of the air thermometer being −273° C.

(6) Determine the height of the barometer when a milligramme of air at 27° C. occupies a volume of 20 c.cm. in a tube over mercury, the mercury standing 73 cm. higher inside the tube than outside. (1 gram of air at 0° C. under a pressure of 76 cm. of mercury measures 773·4 c.cm.)

(7) The volume of a bubble of gas, generated under water at a depth of 100 metres, is 2 c.cm. Find its volume when it reaches the surface, assuming the temperature to increase 1° C. for each 10 metres rise, the temperature at the surface to be 15° C., and the pressure of the air to be equal to that due to 10 metres of water.

(8) A litre of hydrogen weighs 0·0896 gram at 0° C. and 760 mm. pressure. Find the weight of a litre at 20° C. and 766 mm. pressure.

(9) Compare the density of air at 10° C. and 750 mm. pressure with its density at 15° C. and 760 mm. pressure.

(10) A flask containing air is corked up at 20° C. and heated in an air bath. A pressure of two atmospheres inside the flask will force the cork out; at what temperature will this take place?

(11) At what temperature will a volume of air at 0° C., when heated at constant pressure, double its volume?

(12) A thin spherical glass bulb 2 cm. in diameter, containing air, is sealed up and enclosed in a spherical vessel 10 cm. in diameter, and containing the same quantity of air as the bulb. The temperature in both vessels is then raised until the inner one bursts. The pressure in the enclosing vessel is then found to be 1·5 atmospheres; find the pressure within the bulb just before bursting.

(13) Five hundred cubic centimetres of Oxygen gas are measured when the temperature is 20° C., and the temperature is then raised to 40° C., the pressure meanwhile remaining invariable. What is the volume of the Oxygen at the latter temperature? (The Coefficient of Expansion of Oxygen is $\frac{11}{3000}$.)

(14) A thousand cubic inches of air at a temperature of 30° C. are cooled down to zero, and at the same time the external pressure upon the air is doubled. What is the volume reduced to, the coefficient of expansion of air for 1° C. being 0·00366?

(15) A piece of iron, measuring 1000 c.cm., is weighed at 0° C., and again at 100° C. What will be its apparent change in weight?

Coefficient of expansion of air	. .	= 0·00367;
,, ,, iron (linear)		= 0·000012;
Mass of 1000 c.cm. of air at 0° C. .	.	= 1·293 grams.

The pressure is supposed to be normal throughout.

(16) At the sea-level the barometer stands at 750 mm., and the temperature is 7° C., while on the top of a mountain the barometer stands at 400 mm., and the temperature is − 13° C.; compare the weights of a cubic metre of air in the two places.

These barometric readings may be taken as corrected for temperature.

(17) At what pressure will 1 gram of dry air at 0° C. occupy 10 cubic cm.?

(18) Calculate the weight of dry air in a room 20 × 10 × 2 metres when the barometer is at 77 cm. and the thermometer at 15° C. If the barometer were at 42 cm. and the thermometer at 25° C., what weight of air would the room contain? (1 c.cm. of air at 0° C. and 76 cm. pressure = ·001293 gram).

(19) A barometer tube is 35 inches long. A small quantity of air is left in the tube above the mercury, and the latter stands at 29·9 inches when the true barometer is 30·01 inches. What will be the true barometric height when this barometer reads 28·5 inches?

(20) A certain mass of gas is enclosed in a cylinder fitted with an air-tight piston. The area of the piston is 10 sq. cm., the barometer reads 750 mm. and the thermometer 17° C. A weight of 5 kilogrammes is put on the piston, which causes it to sink. Find to what temperature the cylinder must be heated to make the piston rise to its original position (density of mercury = 13·6 grams per c.c.).

CHAPTER V.

SPECIFIC HEAT.

Units of Heat.—The **calorie** or **gram-degree-Centigrade unit** is used in scientific work: it is the heat required to raise the temperature of one gram of water 1° C. Frequently a unit 1,000 times as large is used, called a *major* or *great calorie*.

Another unit used by British engineers is the **British Thermal Unit** (B.Th.U.), or **pound-degree-Fahrenheit unit:** it is the heat required to raise one pound of water 1° F. To convert from calories to B.Th.U. multiply the calories by $\frac{1}{252}$.

A further unit used by engineers is the **pound-degree-Centigrade unit,** *i.e.* the heat required to raise one pound of water 1° C. (Some writers call this unit the *therm.*)

Accurate definitions specify certain temperatures; these, however, are not required for calculation purposes.

Note that the number giving the specific heat of a body is the same for all three systems. The specific heat of copper is ·092: thus ·092 calorie will raise the temperature of 1 gram of copper 1° C., ·092 B.Th.U. will raise 1 pound of copper 1° F., and ·092 pound-degree-Centigrade unit will raise 1 pound of copper 1° C.

Formulae for Calculations.—(1) If Q denote the quantity of heat lost by a body of mass m and specific heat s in falling in temperature from T° to θ°, then:—

$$Q = ms\,(T - \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Similarly, if the temperature of a body be raised from t° to θ°, the heat (Q_1) gained by the body is given by:—

$$Q_1 = ms\,(\theta - t).$$

(2) If w denote the water equivalent of a body of mass m and specific heat s,

$$w = ms \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Method of Mixtures.—The principle of calculation is briefly as follows:—The method involves the loss of heat by one portion of the system, and a corresponding gain of heat by the remaining portion. If the former portion be represented by A, and the latter by B, the principle is:—

$$\left.\begin{array}{l}\text{The loss of heat by A}\\ \quad\text{during the change}\\ \quad\text{from its initial to its}\\ \quad\text{final state}\end{array}\right\} = \left\{\begin{array}{l}\text{The gain of heat by B}\\ \quad\text{during the change}\\ \quad\text{from its initial to its}\\ \quad\text{final state.}\end{array}\right.$$

In general symbols, if m_1 be the mass of a solid say, s its specific heat, and T its initial temperature: m_2 the mass of the water say, and t its initial temperature: w the water equivalent of the calorimeter, etc.: θ the final temperature, then:—

$$m_1 s\,(\mathrm{T} - \theta) = (m_2 + w)\,(\theta - t) \quad\ldots\ldots\ldots\ldots\ (3)$$

In applying the above the following points must be attended to: (a) There must be no loss of heat to, or gain of heat from, the outside. (b) The total mass of the system must be constant throughout: this condition is really involved in (a).

Method of Cooling (Liquids).—Let t_1 = *number of seconds* taken by the liquid to cool from T° to $\theta°$, m_1 = mass of the liquid, t_2 = number of seconds taken by the water to cool from T° to $\theta°$, m_2 = mass of the water, w = water equivalent of the calorimeter and thermometer, s = specific heat of the liquid, and K = a constant depending on the radiating surface, then:—

H_1 = Heat given out by liquid, etc., per second

$$= \mathrm{K}\,\frac{m_1 s\,(\mathrm{T} - \theta) + w\,(\mathrm{T} - \theta)}{t_1} = \mathrm{K}\,\frac{(m_1 s + w)(\mathrm{T} - \theta)}{t_1}.$$

H_2 = Heat given out by the water, etc., per second,

$$= \mathrm{K}\,\frac{(m_2 + w)\,(\mathrm{T} - \theta)}{t_2}.$$

But
$$\mathrm{H}_1 = \mathrm{H}_2$$

$$\therefore\ \frac{m_1 s + w}{m_2 + w} = \frac{t_1}{t_2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4)$$

from which s is determined. Note that the calorimeter is filled *to the same height* with the water and with the liquid.

Electrical Method (*Liquids*) :—The heat developed in t seconds is $\cdot 24\text{EI}t$ calories, where I is the current in amperes, and E the pressure at the terminals in volts. Equating this to the heat taken up by m grm. of the liquid of specific heat s, and by the apparatus (water equivalent w grm.) we get :—

$$ms(\text{T} - \theta) + w(\text{T} - \theta) = \cdot 24\text{EI}t \quad \dots\dots\dots\dots\dots\dots (5)$$

from which s is determined. T° C. = final, and $\theta°$ C. = original temperature.

Method of Fusion of Ice and Condensation Method :—Calculations on these involve the idea of latent heat, and are dealt with in Chapters VI. and VII.

Gases :—Gases have two specific heats, viz. the specific heat at constant pressure (s_p) and the specific heat at constant volume (s_v), the former being the greater owing to the external work done by the gas in expanding. For dry air $s_p = \cdot 2375$ and $s_v = \cdot 1684$, so that the ratio $\gamma = s_p/s_v = 1 \cdot 4$.

In *Clement and Desormes'* method of finding the ratio of the specific heats the formula for calculations is :—

$$\gamma = \frac{s_p}{s_v} = \frac{\log \text{P} - \log p_1}{\log p_2 - \log p_1} = \frac{\text{P} - p_1}{p_2 - p_1} \text{ (approx.)} \quad \dots\dots\dots (6)$$

where p_1 = initial pressure, p_2 = final pressure, and P = atmospheric pressure. (See Example (10)).

Again, the *velocity of sound* (v) in a gas, *e.g.* in air, is given by $v = \sqrt{\gamma \frac{\text{P}}{\text{D}}}$, where P = pressure, and D = density : hence—

$$\gamma = v^2 \frac{\text{D}}{\text{P}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

an expression from which the ratio (γ) of the specific heats can be determined. (See Example (11)).

Vapours :—It may be noted that the specific heat of *non-saturated steam* is of the order $\cdot 4$ to $\cdot 6$. **The specific heat of saturated steam is negative,** *i.e.* it gives out heat when its temperature is raised.

Worked Examples.—(1) *Ten grams of water at* 98° C. *are poured into a copper vessel weighing* 25 *grams and containing* 100 *grams of water at* 6° *C. Find the final temperature of the mixture. Specific heat of copper* = 0·092.

Here, if θ denote the final temperature, we have—

Heat lost by the ten grams of water initially at 98° C. $= 10(98 - \theta)$ calories.

Heat gained.—(1) By copper vessel $= 25 \times \cdot 092\,(\theta - 6)$ calories $= 2\cdot 3\,(\theta - 6)$ calories.

(2) By water in copper vessel $= 100\,(\theta - 6)$.

Hence, equating we get—

$$10\,(98 - \theta) = 102\cdot 3\,(\theta - 6).$$

$$\therefore\ 112\cdot 3\theta = 1593\cdot 8 \quad \text{or} \quad \theta = 14\cdot 2^\circ\,\text{C. (nearly)}.$$

(2) *One ounce of copper at 212° F. is immersed in 1 lb. of water at 55° F. Find the final temperature. (Specific heat of copper = ·1.)*

Let t° F. be the final temperature.

Heat given out by $\frac{1}{16}$ lb. of copper in cooling from 212° F. to t° F. $= \{\frac{1}{16} \times \cdot 1 \times (212 - t)\}$ B.Th.U. (pound-degree-Fahrenheit).

Heat taken up by 1 lb. of water in rising from 55° F. to t° F. $= 1 \times 1 \times (t - 55) = (t - 55)$ B.Th.U.

$$\therefore\ \frac{212 - t}{16 \times 10} = t - 55$$

$$160t - 8800 = 212 - t,$$

$$\therefore\ t = 56^\circ\ \text{F. nearly}.$$

(3) *In order to determine the specific heat of silver, a piece of the metal, weighing 21 grams, is heated to 98° C. and then dropped into a calorimeter containing 100 grams of water at 10° C. The final temperature of the mixture is 11° C.: find the specific heat of silver. The water equivalent of the calorimetric apparatus is 3·6 grams.*

Here, if s denote the specific heat of silver, we get—

Heat lost by the silver

$$= 21 \times s \times (98 - 11) = 1827s \text{ units}.$$

Heat gained—

(*a*) By calorimetric apparatus $= 3\cdot 6\,(11 - 10) = 3\cdot 6$ units.

(*b*) By water in calorimeter $= 100\,(11 - 10) = 100$ units.

Hence, $1827s = 103 \cdot 6$

$$\therefore s = \frac{103 \cdot 6}{1827} = 0 \cdot 057.$$

(4) *Ten grams of common salt at 91° C. having been immersed in 125 grams of oil of turpentine* at 13° C., the temperature of the mixture was 16° C. Find, from these data, the specific heat of common salt, supposing no loss or gain of heat to have taken place from without and taking the specific heat of oil of turpentine as* 0·428.

Here we have—

$$10 \times s \times (91 - 16) = 125 \times 0 \cdot 428 \times (16 - 13)$$

$$\therefore 750s = 160 \cdot 5$$

$$\therefore s = \frac{160 \cdot 5}{750} = 0 \cdot 214.$$

(5) *A mass of* 200 *grams of copper, whose specific heat is* 0·095, *is heated to* 100° *C., and placed in* 100 *grams of alcohol at* 8° *C., contained in a copper calorimeter whose mass is* 25 *grams, and the temperature rises to* 28·5° *C. Find the specific heat of the alcohol.*

The heat lost by the copper

$$= 200 \times \cdot 095 \times (100 - 28 \cdot 5)$$
$$= (200 \times \cdot 095 \times 71 \cdot 5) \text{ calories.}$$

The heat gained—

(*a*) By the calorimeter $= 25 \times \cdot 095 \times (28 \cdot 5 - 8)$
$= (25 \times \cdot 095 \times 20 \cdot 5)$ calories.

(*b*) By the alcohol $= (100 \times s \times 20 \cdot 5)$ calories,

where s denotes the specific heat of the alcohol. Then, equating total loss and gain of heat, we have—

$$(200 \times \cdot 095 \times 71 \cdot 5) = (25 \times \cdot 095 \times 20 \cdot 5) + (100 \times 20 \cdot 5 \times s),$$

$$1358 \cdot 5 = 48 \cdot 7 + 2050s.$$

$$\therefore 2050s = 1309 \cdot 8.$$

$$\therefore s = 0 \cdot 639.$$

ter could not be used in this case as common salt is soluble in
This point should be noticed.

(6) *The following data were obtained in an experiment for the determination of the water equivalent of a given calorimetric apparatus :—*

Weight of apparatus	45·623 *grams.*
,, ,, ,, + *water*	224·583 ,,
Initial temperature of apparatus and water ...	9° *C.*
Temperature of hot water	78° *C.*
Final temperature	13·2 *C.*
Weight of apparatus after addition of hot water	236·493 *grams.*

[*Note.—These data are given in the order of their determination in an actual experiment.*]

Here—

Weight of water in calorimeter = 224·583 − 45·623 = 178·960 grams, and weight of hot water added

$$= 236{\cdot}493 - 224{\cdot}583 = 11{\cdot}91 \text{ grams.}$$

Therefore, if w denote the water equivalent of the apparatus, we have—

$$(178{\cdot}96 + w)\,(13{\cdot}2 - 9) = 11{\cdot}91\,(78 - 13{\cdot}2),$$

$$4{\cdot}2\,(178{\cdot}96 + w) = 11{\cdot}91 \times 64{\cdot}8,$$

$$751{\cdot}632 + 4{\cdot}2w = 771{\cdot}768,$$

$$\therefore\ 4{\cdot}2w = 20{\cdot}136,$$

$$\therefore\ w = 4{\cdot}794 \text{ grams.}$$

(7) *A ball of platinum, whose mass is* 200 *grams, is removed from a furnace and immersed in* 150 *grams of water at* 0° *C. If we suppose the water to gain all the heat which the platinum loses, and if the temperature of the water rises to* 30° *C., what is the temperature of the furnace? Specific heat of platinum is* 0·031.

Here, if T° C. denote the temperature of the furnace, we have—

$$200 \times {\cdot}031\,(T - 30) = 150 \times 30.$$

$$\therefore\ 6{\cdot}2\,(T - 30) = 4500.$$

$$\therefore\ 6{\cdot}2T = 4500 + 186 = 4686.$$

$$\therefore\ T = \frac{4686}{6{\cdot}2} = 756^\circ \text{ C.}$$

[This example indicates a method of measuring very high temperatures.]

(8) *A volume of air, at* 100° *C., whose mass is* 30 *grams, is passed through a copper worm, immersed in* 197 *grams of water initially at* 10° *C., and finally at* 13° *C. Find the specific heat of air, given that the water equivalent of the calorimetric apparatus is* 10 *grams, and that the loss of heat by cooling of the calorimeter during the experiment is* 9 *gram-degree units.*

Here the air is not all cooled to the same temperature—that which passes through first is cooled down to 10° C., but as the temperature of the calorimeter rises the fall of temperature of the air becomes gradually less. If the increase of temperature in the calorimeter be supposed to be uniform, the heat actually lost by the air is approximately equal to that given by supposing the total mass of air to be cooled down to the *mean* temperature of the calorimeter during the time occupied by the experiment. Hence the solution is as follows :—

Heat lost by the air

$$= 30 \times s\left(100 - \frac{10 + 13}{2}\right) \text{ units} = 2655s \text{ units.}$$

Heat gained by calorimeter and contained water

$$= 3\,(197 + 10) = 207 \times 3 = 621 \text{ units.}$$

The heat lost by the cooling of the calorimeter was originally derived from the air,

$$\therefore \text{ the total gain of heat} = 621 + 9 = 630 \text{ units.}$$

Hence, equating we get

$$2655s = 630$$

$$\therefore\ s = \frac{630}{2655} = 0{\cdot}237.$$

This example gives *in outline* the method of determining the specific heat of a gas at constant pressure. Note particularly that the air passes *through* the calorimeter, and does not remain in it.

(9) *In a determination of the specific heat of a liquid by the method of cooling, the weight of calorimeter (copper)* = 16·24 *grams; its weight containing liquid* = 27·18 *grams, and containing water* = 30·14 *grams. The times of cooling from* 60° *C. to* 55° *C. were* 140 *secs. with liquid and* 330 *secs. with water. Specific heat of copper is* ·095. *Find the specific heat of the liquid.*

Calories lost per second in the water experiment when cooling from 60° C. to 55° C.

$$=\frac{(13{\cdot}9+16{\cdot}24\times{\cdot}095)5}{330}.$$

Calories lost per second in the liquid experiment when cooling from 60° C. to 55° C.

$$=\frac{(10{\cdot}94s+16{\cdot}24\times{\cdot}095)5}{140}.$$

Equating these two, cancelling and cross-multiplying we get:—

$$14\,(13{\cdot}9+1{\cdot}54)=33\,(10{\cdot}94s+1{\cdot}54)$$

i.e. $\quad 10{\cdot}94s=\frac{14\times 15{\cdot}44}{33}-1{\cdot}54=6{\cdot}55-1{\cdot}54=5{\cdot}01.$

$$\therefore\ s={\cdot}458={\cdot}46 \text{ very nearly.}$$

(10) *In an experiment on air by Clement and Desormes' method the initial pressure was* 986·4 *units, the final pressure* 1015·2 *units. The atmospheric pressure was* 1027 *units. Find the ratio of the specific heats of air.*

$$\gamma=\frac{s_p}{s_v}=\frac{\log 1027-\log 986{\cdot}4}{\log 1015{\cdot}2-\log 986{\cdot}4}=\frac{3{\cdot}0116-2{\cdot}9941}{3{\cdot}0065-2{\cdot}9941}.$$

$$\therefore\ \gamma=\frac{{\cdot}0175}{{\cdot}0124}=1{\cdot}411,$$

or $\quad \gamma=\frac{1027-986{\cdot}4}{1015{\cdot}2-986{\cdot}4}=\frac{40{\cdot}6}{28{\cdot}8}=1{\cdot}41$ nearly.

(11) *The velocity of sound in air at* 0° *C. and* 76 *cm. pressure is* 33180 *cm. per sec. The specific heat of air at constant pressure is* ·2375. *Find the specific heat at constant volume.*

$$\gamma=v^2\frac{D}{P}=(33180)^2\times\frac{{\cdot}001293}{76\times 13{\cdot}6\times 981}=1{\cdot}404.$$

i.e. $\quad \frac{s_p}{s_v}=1{\cdot}404 \quad \therefore\ s_v=\frac{s_p}{1{\cdot}404}.$

Hence:— $\quad s_v=\frac{{\cdot}2375}{1{\cdot}404}={\cdot}162.$

Note that 1 c.cm. of air at 0° C. and 76 cm. pressure = ·001293 grms. = D, and P = 76 × 13·6 × 981 dynes per sq.cm. Special attention must be paid to the units.

Exercises V.

(1) 280 grams of zinc (specific heat = ·095) are raised to the temperature 97°C. and immersed in 150 grams of water at 14°C. contained in a copper calorimeter weighing 96 grams. The specific heat of copper being ·095, what will be the temperature of the mixture supposing that there is no exchange of heat except among the substances mentioned? What is the water equivalent of the calorimeter employed?

(2) A copper vessel containing a thermometer is at 12° C.; 50 grams of water at 60° C. are poured in, and the temperature, after stirring, is found to be 50° C.: find the thermal capacity, or water equivalent, of the vessel and thermometer.

(3) In a determination of the specific heat of a liquid by the method of cooling the following data were obtained:—

Weight of calorimeter (copper)	16·24 grams.
,, ,, and liquid	27·18 ,,
,, ,, ,, water...	30·14 ,,
Time of cooling of liquid from 60° C. to 55° C....	140 seconds.
,, ,, water ,, ,, ...	330 ,,

Find the specific heat of the liquid. [The water equivalent of the calorimeter must be determined from its weight and the specific heat of its material.]

(4) Determine the specific heat of copper from the following data:—

Weight of copper	16·65 grams.
,, water in calorimeter	49 ,,
Initial temperature of copper	99·5° C.
,, ,, water and calorimeter ...	12° C.
Final ,, mixture	14·5° C.
Water equivalent of calorimeter, etc.	2·1 grams.

(5) Determine the specific heat of alcohol from the following data:—

Weight of copper calorimeter	20·4 grams.
,, ,, ,, + alcohol	70·5 ,,
,, ,, dropped into calorimeter ...	10·5 ,,
Initial temperature of calorimeter and alcohol ...	10° C.
,, ,, ,, copper	98° C.
Final ,, ,, mixture	12·6° C.

(6) Ten grams of sulphuric acid, enclosed in a sealed glass tube weighing 4·3 grams, are heated to 80° C. and dropped into 86 grams of water at 10° C. contained in a copper vessel weighing 15 grams. Find the final temperature of the mixture, the specific heat of sulphuric acid being taken as 0·34.

(7) A piece of platinum, weighing 120 grams, is taken from a furnace and at once dropped into 100 grams of water at 10° C., contained in a copper vessel weighing 21 grams. The final temperature is found to be 37° C.: find the temperature of the furnace.

(8) 100 grams of mercury at 250° C. are mixed with 80 grams of mercury at 15° C. in a glass vessel weighing 35 grams. Find the final temperature of the mixture.

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(9) Regnault found that 100·5 units of heat were required to raise the temperature of unit mass of water from 0° C. to 100° C., and 203·2 units to raise its temperature to 200° C. Find the mean specific heat of water between 0° C. and 100° C., between 100° C. and 200° C., and between 0° C. and 200° C.

(10) Equal weights of two liquids of specific heats s' and s'', at temperatures t' and t'', are poured into a glass vessel of mass m, specific heat s, and temperature t. Find the final temperature of the mixture.

(11) Equal volumes of two liquids A and B are mixed. Find the final temperature.

Specific gravity of A = 1·8; of B = ·56.
Specific heat of A = 0·3; of B = 0·6
Temperature of A = 60° C.; of B = 40° C.

(12) One hundred litres of hydrogen, measured at 0° C., are heated in an oil bath to 210° C. and then passed through a calorimeter containing 500 grams of water initially at 10° C., and finally at 21·75° C. Find the specific heat of hydrogen, given that the water equivalent of the calorimetric apparatus = 5 grams, loss of heat by cooling of calorimeter during passage of gas = 3·6 gram-degree units, weight of 1 litre of hydrogen = 0·0896 grams at 0° C.

CHAPTER VI.

CHANGE OF STATE:—LIQUEFACTION AND SOLIDIFICATION: LATENT HEAT OF FUSION.

Formulae for Calculations.—(1) If L denote the latent heat of fusion of a given substance, then the quantity of heat **absorbed during fusion** by a mass m of that substance is represented by mL, and the quantity of heat **evolved during solidification** of a mass m of the substance is also represented by mL.

[In each of these cases the temperature of the substance remains constant during the change of state, but the heat absorbed, or given out, affects the temperature of adjacent substances.]

✓ (2) *Latent Heat of Water* (*Fusion of Ice*). Let M be the mass of water in the calorimeter, t its temperature, w the water equivalent of the calorimeter, etc., θ the final temperature of the water after the melting of the ice, and m the mass of ice added. Then

Heat lost by water and calorimeter $= (\mathrm{M} + w)(t - \theta)$.

Heat absorbed by ice in changing from ice at 0° C. to water at 0° C. $\Big\} = m\mathrm{L}$.

Heat gained by water thus produced in being raised from 0° C. to θ° C. $\Big\} = m\theta$.

$$\therefore\ (\mathrm{M} + w)(t - \theta) = m\mathrm{L} + m\theta,$$

$$i.e.\quad \mathrm{L} = \frac{(\mathrm{M} + w)(t - \theta) - m\theta}{m}.$$

[It is not necessary to remember the formula: problems can be worked out from first principles by the method here indicated.]

(3) *Ice Calorimeters and the Determination of Specific Heats.* (*a*) *Laplace and Lavoisier:* If M denote the mass of the solid, the specific heat of which (s) is required, and T° C. its temperature, and if when placed in the receptacle m is the mass of ice melted, we have :—

Heat given out by solid in cooling from T° C. to 0° C. $= MsT$.

Heat absorbed by ice in melting $= mL$.

$$\therefore MsT = mL,$$

$$i.e. \quad s = \frac{mL}{MT}.$$

Knowing L, the specific heat is determined.

(*b*) *Bunsen.*—Let M be the mass in grm. of the substance of which the specific heat (s) is required, T° C. its temperature, v the diminution in volume in c.cm. Now

Decrease in vol. when 1 grm. of ice melts $= \cdot 09068$ c.cm.

$$\therefore \text{ Mass of ice melted} = \frac{v}{\cdot 09068} \text{ grm.} = 11v \text{ approx.}$$

Further :—

Heat given out by the solid $= MsT$.

Heat required to melt the ice $= 80 \times 11v = 880v$.

$$\therefore MsT = 880v,$$

$$i.e. \quad s = \frac{880v}{MT}.$$

[It is not necessary to remember the formula : problems can invariably be worked out from first principles from the data given.]

Worked Examples. (1) *Ten grams of ice at* $-10°$ *C. are mixed with* 120 *grams of water at* 80° *C. Find the final temperature of the mixture.* (*Specific heat of ice* $= 0{\cdot}5$ *and latent heat of water* $= 80$.)

Here, if θ denote the final temperature we have—

Loss of heat by water $= 120\,(80 - \theta)$.

Gain of heat by—

(*a*) Ice during change of temperature from $-10°$ C., to 0° C. $= (10 \times \cdot 5 \times 10)$ units $= 50$ units.

(4) Determine the latent heat of ice from the following data.

Weight of brass calorimeter (Sp. heat ·09) ...	35 gm.
,, ,, ,, ,, + water	156 gm.
Initial temperature of water and calorimeter	24° C.
Final ,, ,, ,, ...	17° C.
Weight of calorimeter, etc., after addition of ice	165 gm.

(5) A gram of ice at 0° C. contracts 0·091 c.cm. in becoming water at 0° C. A piece of metal weighing 10 grams is heated to 50° C. and then dropped into the calorimeter. The total contraction is ·063 c.cm.: find the specific heat of the metal, taking the latent heat of ice as 80.

(6) Five hundred cubic centimetres of mercury at 56° C. are put into a hollow in a block of ice and it is found that 159 grams of ice are liquefied; find the specific heat of mercury.

(7) Ten grams of water at 96° C. are placed in the inner tube of a Bunsen's calorimeter, and it is found that the volume of the contents of the outer portion decreases by 1·09 c.cm.: taking the latent heat of water as 80 what value does this give for the specific gravity of ice?

(8) The latent heat of fusion of ice is 79·5. The specific gravity is ·917. Ten grams of metal at 100° C. are immersed in a mixture of ice and water, and the volume of the mixture is found to be reduced by 125 c.mm., without change of temperature. Find the specific heat of the metal.

(9) The specific heat of iron is ·113; how many lb. of iron at 250° C. must be introduced into an ice calorimeter in order to produce 2 lb. of water?

(10) If 100 c.cm. of water in freezing become 109 c.cm. of ice, and the introduction of 20 grams of mercury at 100° C., into a Bunsen's calorimeter causes the end of the column of mercury to move through 74 mm. in a tube 1 sq. mm. in section, find the specific heat of mercury. (The heat required to melt one gram of ice is 80 units.)

(11) A pound of ice at 0° C. is thrown into 6 lb. of water at 15° C. contained in a copper vessel weighing 3 lb. and when the ice is melted the temperature of the water is 2° C. Find the latent heat of fusion of ice, the specific heat of copper being 0·095.

(12) Define the terms latent heat, specific heat, and capacity for heat. The specific heat of copper is ·095. What is the capacity for heat of 500 grams of copper? If 500 grams of copper are heated to 100° C., and placed in an ice calorimeter, how much ice is melted, the latent heat of fusion of ice being 80?

CHAPTER VII.

CHANGE OF STATE:—VAPORISATION AND CONDENSATION: LATENT HEAT OF VAPORISATION.

Formulae for Calculations.— (1) *Dalton's Second Law.* If p_1, p_2, p_3 denote the individual pressures due to the several constituents of a mixture of vapours then we have $P = p_1 + p_2 + p_3$, where P is the total pressure exerted by the mixture. The most familiar example of the application of this law is in the case of air and water vapour. If P′ denote the pressure due to the air alone, and f that due to the vapour, then the total pressure, P, is given by $P = P' + f$.

(2) *Mass of a Given Volume of Moist Air.*—The density of water vapour compared with air is ·62 *or* 5/8. Thus in say 1 litre (1000 c.cm.) of moist air at temperature $t°$ C. and *total pressure* P, the air containing vapour at pressure f, we have:—

(*a*) 1 lit. of dry air at $t°$ C. and pressure $P - f$ weighing:—

$$1{\cdot}293 \times \frac{273}{273 + t} \times \frac{P - f}{760} \text{ grams.} \quad \text{(See Chapter IV.)}$$

(*b*) 1 lit. of water vapour at $t°$ C. and pressure f weighing:—

$$\cdot 62 \times 1{\cdot}293 \times \frac{273}{273 + t} \times \frac{f}{760} \text{ grams,}$$

and the total weight of the litre of moist air is (*a*) + (*b*).

(3) *"Total Heat" and "Latent Heat" of Steam at T° C.*— If Q be the total heat required to raise unit mass of water from 0° C. to T° C. and to convert it into steam at T° C., then:—

$$Q = \text{Total Heat of Steam at T° C.}$$
$$= 606{\cdot}5 + \cdot 305T \ldots\ldots\ldots\ldots (1)$$

If the mass be one gram the above gives Q in calories, if it be one pound Q will be in pound-degree-Centigrade units.

If L be the latent heat at T° C., then $Q = T + L$, for T units are absorbed by unit mass of water in rising from 0° C. to T° C. Hence—

$$L = Q - T = \text{Latent Heat of Steam at T° C.}$$
$$= 606{\cdot}5 - {\cdot}695T \quad \text{.......... (2)}$$

If B.Th.U. are employed the above expressions become:—

$$Q = 1082 + {\cdot}305t \quad \text{.......... (3)}$$
$$L = 1114 - {\cdot}695t \quad \text{.......... (4)}$$

$t°$ being, of course, $t°$ F. (Here Q = B.Th.U. required to raise 1lb. of water from 32° F. to $t°$ F. and to convert it into steam at $t°$ F.).

Heat given to a substance which produces a rise in temperature is called *Sensible Heat.* **Total Heat = Sensible Heat + Latent Heat.**

Internal Heat of Evaporation is the heat equivalent of the work done in altering the molecular condition of the substance in passing from liquid to vapour. *External Heat of Evaporation* is the heat equivalent of the external work done in overcoming the atmospheric pressure as the substance increases in volume in passing from liquid to vapour. **Latent Heat = Internal Heat + External Heat.**

It may be noted that a given volume of water when converted into steam under atmospheric pressure occupies a volume nearly 1600 times as great.

If in 1lb. of *wet* steam we find Q pound of steam and (1 − Q) pound of water, Q is the *dryness fraction* of the steam.

Determination of Latent Heat of Vaporisation.—If L denote the latent heat of vaporisation of a given liquid then the quantity of **heat absorbed during vaporisation** by a mass m of that liquid is denoted by mL, and the quantity of **heat evolved during condensation** of a mass m of the vapour is also denoted by mL.

In calculations on this, say on the latent heat of steam, the reasoning is as follows:—If M = mass of water in calorimeter, w = water equivalent, t = initial temperature of water and calorimeter, T = temperature of steam, m = mass of

steam condensed, θ = final temperature, and L = latent heat of steam, then :—

Heat lost by steam at T° in changing to water at T° = mL.

Heat lost by this water in cooling to $\theta°$ = m (T − θ).

Heat gained by calorimeter and water in rising in temperature from $t°$ to $\theta°$ = (M + w) (θ − t).

$$\therefore\ mL + m\,(T - \theta) = (M + w)\,(\theta - t)$$

$$\therefore\ L = \frac{(M + w)\,(\theta - t)}{m} - (T - \theta) \quad \ldots\ldots (5)$$

[The formula need not be used in problems: the latter should be worked out from first principles on the lines indicated above.]

Hypsometry.—The difference of level x (cm.) between two stations A and B where the atmospheric pressures are P_1 and P_2 (cm.) respectively is given by :—

$$x = 1840000\,(\log P_1 - \log P_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Knowing the temperatures at which water boils at the two stations, the pressures P_1 and P_2 are obtained from a table of vapour pressures, and thus x is determined.

Worked Examples.—(1) *Find the latent heat of steam near* 100° *C. from the following data—*

Weight of calorimeter	105 *gm.*
,, ,, ,, *and water*	346 ,,
Initial temperature of calorimeter and water	4° C.
Final ,, ,, ,, ,, ,,	24° C.
Weight of calorimeter, etc., at the end of the experiment	354·16 *gm.*
Water equivalent of calorimetric apparatus	9 *gm.*
Height of barometer	752 *mm.*

(*The steam is produced at atmospheric pressure*).

Here, from data of question we have—

Weight of water in the calorimeter = 346 − 105 = 241 grams;

Weight of steam condensed = 354·16 − 346 = 8·16 grams;

Temperature of the steam = 99·7° C.

Hence the **loss of heat**—

(*a*) By the steam during condensation $= m\text{L} = 8{\cdot}16$ L calories.

(*b*) By the water, produced on condensation of the steam, in cooling from $99{\cdot}7°$ C. to $24°$ C. $= 8{\cdot}16\ (99{\cdot}7 - 24)$ $= 8{\cdot}16\ (75{\cdot}7) = 618$ calories (approx.);
and, the **gain of heat** by the calorimetric apparatus

$$= (241 + 9)\ (24 - 4) = 250 \times 20 = 5{,}000 \text{ calories.}$$

Therefore :— $\quad 8{\cdot}16\,\text{L} + 618 = 5{,}000.$

$$\therefore\ 8{\cdot}16\,\text{L} = 4382, \textit{ i.e. } \text{L} = 537.$$

(2) *A copper vessel, weighing* 100 *grams, contains* 300 *grams of water at* 0° *C., and* 50 *grams of ice at* 0° *C. Find the quantity of steam, at* 100° *C., that must be blown into the vessel, to raise its temperature and that of its contents to* 10° *C. (Sp. heat of copper* = 0·095 ; *latent heat of steam* = 537 ; *latent heat of water* = 80.)

Let m denote the mass of the necessary quantity of steam. Then the **heat lost**—

(*a*) By the steam during condensation $= 537\,m$ calories.

(*b*) By water so produced in cooling from 100° C. to 10° C.

$$= m\ (100 - 10) = 90\,m \text{ calories}$$

and, the **heat gained**—

(*a*) By the copper vessel $= 100 \times {\cdot}095 \times 10 = 95$ calories.

(*b*) „ „ water in the vessel $= 300 \times 10 = 3{,}000$ calories.

(*c*) „ „ ice during liquefaction $= 50 \times 80 = 4{,}000$ „

(*d*) „ „ water, produced by the liquefaction of the ice, in being raised from 0° C. to 10° C. $= 50 \times 10 = 500$ calories.

Hence, equating we get—

$$627m = 7595.$$

$$\therefore\ m = 12{\cdot}1 \text{ grams.}$$

(3) *Steam calorimeter for the determination of specific heat :—A piece of metal of mass* 200 *gm. and temperature* 20° *C. is suspended in a steam calorimeter and* 1·5 *gm. of steam are found to condense on it. Find the specific heat of the metal.*

Taking the latent heat of steam as 540 (approx.) the heat given out by the steam at 100° C. in condensing to water at 100° C. $= 1{\cdot}5 \times 540 = 810$ units.

The heat taken up by the metal in rising in temperature from 20° C. to 100° C. $= 200 \times s \times 80$ units $= 16000\,s$ units.

$$\therefore\ 16000\,s = 810,$$

i.e. $$s = {\cdot}05.$$

(4) *A quantity of hydrogen is collected, over water, in a eudiometer tube. The height of the column of water left in the tube is* 40·8 *mm., and its temperature is* 15° *C.; find the pressure of the hydrogen in the upper part of the tube. (Take the height of the barometer as* 758 *mm.).*

The space occupied by the hydrogen is saturated with aqueous vapour. Hence, with the notation used above—

$$P = P' + f,$$
$$\therefore\ P' = P - f,$$

where P denotes the total pressure in the tube, P′ that due to the hydrogen, and f that due to the aqueous vapour present.

But since 40·8 mm. of water are equivalent to $\frac{40{\cdot}8}{13{\cdot}6} = 3$ mm. of mercury we have that—

$$P = 758 - 3 = 755 \text{ mm.}$$

and, from a table of vapour pressures, we get—

$$f = 12{\cdot}7 \text{ mm.}$$
$$\therefore\ P' = 755 - 12{\cdot}7 = 742{\cdot}3 \text{ mm.}$$

(5) *A mixture of air and of the vapour of a liquid in contact with excess of the liquid is contained in a vessel of constant volume. At a temperature of* 15° *C. the pressure in the vessel is* 70 *cm. of mercury, at* 30° *C. it is* 88 *cm., at* 45° *C. it is* 110 *cm., and at* 60° *C. it is* 145 *cm. Assuming that at* 15° *C. the vapour pressure of the liquid is* 15·4 *cm. calculate the vapour pressures at* 30°, 45°, *and* 60° *C.*

At 15° C. the pressure is 70 cm., but that due to the vapour is 15·4 cm.

$$\therefore \text{ that due to the air is } 70 - 15{\cdot}4 = 54{\cdot}6 \text{ cm.}$$

Hence the pressure due to the air at 30° C. is

$$\left[\text{from } \frac{P_{30}}{P_{15}} = \frac{T_{30}}{T_{15}}\right]:—$$

$$\frac{54{\cdot}6 \times (273 + 30)}{273 + 15} = \frac{54{\cdot}6 \times 303}{288} = 57{\cdot}4 \text{ cm.}$$

Similarly the air pressures at 45° C. and 60° C. are 60·3, and 63·2 cm. respectively.

Deducting the air pressures from the corresponding total pressures 88, 110, 145 we get 30·6 cm., 49·7 cm., and 81·8 cm. for the respective vapour pressures.

(6) *A barometer tube dipping into a mercury reservoir contains a mixture of air and saturated vapour above a column of mercury which is 70 cm. above the level in the reservoir, the atmospheric pressure being 76 cm. What is the height of the mercury column when the tube is depressed so as to reduce the volume occupied by the air to one-half of its original value, the pressure of the saturated vapour being 1·5 cm.?*

The pressure of the air and saturated vapour to begin with is 6 cm.; therefore the pressure of the air = 6 — 1·5 cm. = 4·5 cm When the volume of the air is reduced to one-half of its original value the air pressure will = 9·0 cm. Add to this the vapour pressure of 1·5 cm. and we get a total pressure of 10·5 cm., whence the height of the mercury column = 76 — 10·5 = 65·5 cm.

(7) *On the top of a certain mountain water boils at a temperature of 93° C. Find the height above sea level.*

On reference to a table of vapour pressures, it will be seen that if water boils at 93° C. the pressure of the atmosphere is 58·8 cm. Then

$$\begin{aligned} x &= 1840000 (\log P_1 - \log P_2) \\ &= 1840000 (\log 76 - \log 58{\cdot}8) \\ &= 1840000 \times {\cdot}1114 \\ &= 205000 \text{ cm.} \\ &= 6725{\cdot}7 \text{ feet.} \end{aligned}$$

(8) ***Find the total heat of 1 lb. of wet steam in which the dryness fraction is ·9, the boiling point being 212° F. and the latent heat of steam 967 B.Th.U. at 212° F.***

(*a*) *Sensible Heat.* To raise 1 lb. of water from 32° F. to 212° F. requires 180 B.Th.U.

(*b*) *Latent Heat.* ·9 lb. is converted into steam and the heat required for this is (·9 × 967) B.Th.U.

∴ Total Heat of the 1 lb. of *wet steam* = 180 + (·9 × 967)
= 1050 B.Th.U.

or (but more 'roundabout'):—

Total Heat of 1 lb. of steam at 212° F. = 1082 + ·305*t*.
= 1082 + (·305 × 212)
= 1146·7 approx.

But ·1 lb. is not vaporised and this means that ·1 × 967, *i.e.* 96·7 must be *subtracted* from the above.

∴ Total Heat of the 1 lb. of *wet steam* = 1050 B.Th.U.

Exercises VII.

(1) When 10 grams of steam at 100° C. were condensed in 1000 grams of water at 0° C., the resulting temperature was 6·3° C. Find the latent heat of steam.

(2) How many grams of steam at 100° C. must be condensed in 300 grams of ice-cold water to raise it to the boiling point?

(3) Calculate the latent heat of vaporisation of water from the following data :—

Mass of calorimeter and thermometer	= 40·45 grams,
Water equivalent of same	= 4·04 ,,
Mass of cold water in calorimeter	= 157·9 ,,
Initial temperature	= 10·6° C.
Final temperature	= 46° C.
Pressure	= 78·168 cms.
Boiling point of water at that pressure	= 100·8° C.
Mass of steam used	= 9·7 grams.

(4) How many lb. of copper of specific heat $\frac{1}{8}$, at 200° C. must be dropped into a mixture of 20 lb. ice and 20 lb. water at 0° C., to convert the whole into steam at 100° C., the water equivalent of the vessel being 5 lb.?

(5) Ten grams of steam at 100° C. are blown into 100 grams of a mixture of ice and water at 0° C. The final temperature of the mixture is 5° C. Find the quantity of ice originally in the mixture.

(6) Ten grams of steam at 60° C. are passed into 600 grams of water at 4° C. The final temperature of the mixture is 14·18° C. Find the latent heat of steam at 60° C. Verify your result by Regnault's formula.

[In an experiment such as this some arrangement must be made for keeping the steam at a constant pressure, corresponding to the maximum pressure of aqueous vapour at 60° C. For example, the steam may be condensed in the calorimeter in a thin copper vessel, communicating with a reservoir of air, in which the pressure can be varied at will.]

(7) Fifty grams of steam at 100° C. are passed into a mixture of 100 grams of ice and 200 grams of water at 0° C. Find the rise of temperature produced. The water equivalent of the vessel containing the mixture of water and ice is 15 grams.

(8) Ten grams of ice at − 10° C. and 100 grams of water at 10° C. are mixed in a copper vessel weighing 150 grams. Twenty grams of steam at 100° C. are then passed into the mixture Find the final temperature of the mixture.

(9) Express the latent heats of steam and water in terms of the degree Fahrenheit.

(10) Draw up from Regnault's formula for total and latent heat of steam a table showing the values of these for the temperatures 50° C., 60° C., etc. . . . 150° C. Note how they change (increase or decrease) with rise of temperature.

(11) The boiling point of water on the top of a mountain is found to be 88° C. What pressure would a barometer indicate there? Express the pressure in dynes per sq. cm. What is the height of the mountain?

(12) Find the temperature at which water boils on the top of Snowdon (height 3570 ft.)

(13) A quantity of dry air measures 1,000 c.cm. at 10° C. and 760 mm. pressure. If the same quantity of air is heated to 30° C., and saturated with aqueous vapour at that temperature, what must be the volume of the moist air, in order that the pressure may remain unchanged? (Tension of aqueous vapour at 30° C. = 31·55 mm.)

(14) A bubble containing 0·01293 milligrams of air is formed 136 mm. below the surface of water at 80° C. Find its volume. [1 c.cm. of air weighs 1·293 milligrams at 0° C., and 760 mm. pressure; height of barometer = 750 mm.; remaining datum required may be obtained from a table of vapour pressures).

(15) A bubble of air is formed 68 mm. below the surface of water at 10° C. Find how many bubbles (of air and water vapour) of volume

equal to its own, this bubble may give rise to, when the temperature of the water rises to 90° C. Height of barometer = 760 mm.

(16) Find the quantity of heat required to vaporise 10 grams of alcohol at 78·3° C.

(17) Twenty grams of ether vapour at 35° C. are passed into 100 grams of ether at 0° C. in a copper vessel weighing 12·5 grams. Find the final temperature of the mixture.

(18) Ten grams of melted lead at 332° C. are dropped into a copper vessel surrounded by melting ice. Find the weight of ice melted.

(19) How many grams of water at 0° C. can theoretically be frozen by the evaporation of 1 gram of water at 0° C. ?

(20) It is found by experiment in a room, where the temperature is 15° C., the dew-point 8° C., and the height of the barometer 750 mm , that a quantity of water in a shallow cylindrical vessel, 10 cm. in radius, loses by evaporation 10 grams of water in 24 hours. If the conditions are such that the formula $m = K\frac{S}{P}(F - f)$ is applicable, find the value of the constant, K, in this case, and apply the formula to find the loss by evaporation from the same vessel when the temperature of the room rises to 20° C., the dew-point remaining constant, and the barometric height being 760 mm.

[S = surface area evaporating. F = maximum pressure of water vapour at the temperature of the room. f = pressure of vapour present = maximum pressure of vapour at the dew-point. P = atmospheric pressure.]

CHAPTER VIII.

HYGROMETRY.

Formulae for Calculations.—(1) In calculating the mass of aqueous vapour present in a given volume of air it must be remembered that the total pressure of the mixture is made up of two pressures:—(*a*) the total pressure of the air; (*b*) the pressure of the vapour present. The latter pressure is equal to the maximum pressure of aqueous vapour at the dew-point, and is the pressure to be employed in calculating the required mass.

(2) The **absolute humidity** of the air is the mass of vapour actually present in unit volume of the air.

(3) The **hygrometric quality** of the air is given by the ratio—

$$\text{H.Q.} = \frac{\text{Mass of vapour present in unit volume of air}}{\text{Mass of unit volume of air}}.$$

(4) The **hygrometric state** or **relative humidity** is given by:—

$$\text{R.H.} = \frac{\text{Mass of vapour present in a given vol. of air at } t^\circ}{\text{Mass of vapour required to saturate the air at } t^\circ} = \frac{m}{m'}$$

and

$$\text{R.H.} = \frac{\text{Pressure of the vapour present at } t^\circ}{\text{Maximum pressure of vapour at } t^\circ}$$

$$= \frac{\text{Maximum pressure of vapour at the dew point}}{\text{Maximum pressure of vapour at temp. of air, } t^\circ} = \frac{f}{F}$$

The Chemical Hygrometer.—The following outlines the steps in the calculations for accurate work:—

(*a*) Let V' = vol. (litres) of air in aspirator at t'° C. = vol. of water drawn off, f' = maximum pressure of vapour at t'° C., f = actual pressure of vapour in atmosphere, H = atmospheric pressure, and V = vol. (litres) the air, which is now in the aspirator, occupied in its initial condition at t° C.

Then from the relation $\frac{V_1P_1}{T_1} = \frac{V_2P_2}{T_2}$, *i.e.* $V_1 = V_2 \cdot \frac{P_2}{P_1} \cdot \frac{T_1}{T_2}$, we have—

$$V = V' \times \frac{H - f'}{H - f} \times \frac{273 + t}{273 + t'} \quad \text{............} \quad (a)$$

in which V and f are the unknowns.

(*b*) Let m = vapour deposited in the drying tubes : then :—

$$m = \tfrac{5}{8}\left[V \times 1{\cdot}293 \times \frac{273}{273 + t} \times \frac{f}{760}\right] \quad \text{.........} \quad (b)$$

in which V and f are again the unknowns. Thus from (*a*) and (*b*) f is determined (and also V if required).

Knowing f, the value of the R.H. is found from the ratio f/F, where F = maximum pressure of vapour at the temperature $t°$ C. of the air (from tables). Also the dew point is found from tables, since f = maximum pressure of vapour at the dew point. Further, R.H. could be found from m/m', since m is known, and m', the mass of vapour required to saturate volume V at $t°$ C., is :—

$$\tfrac{5}{8}\left[V \times 1{\cdot}293 \times \frac{273}{273 + t} \times \frac{F}{760}\right] \quad \text{...........} \quad (c)$$

Wet and Dry Bulb Hygrometer.—The dew point and R.H. are found in various ways, some of which are as follows :—

(*a*) Glaisher's Factors.—*Multiply the difference in reading in degrees (Fahrenheit) of the wet and dry bulb thermometers by the factor corresponding to the reading of the dry bulb, and subtract from the reading of the dry bulb : the result is the dew point.* All the rest follows. (See Example 4).

A few of Glaisher's factors are :—

Dry Bulb. F.	Factor.	Dry Bulb. F.	Factor.	Dry Bulb. F.	Factor.
24°	8·5	45°	2·2	65°	1·8
28°	5·6	50°	2·1	75°	1·7
30°	4·6	55°	2·0	85°	1·6
40°	2·5	60°	1·9	100°	1·57

(*b*) Apjohn's Formulae.—Let d = difference of readings of wet and dry bulb thermometers in degrees Fahrenheit, h = barometer reading *in inches*, w = maximum vapour pressure *in inches* corresponding to the temperature of the wet bulb (this is got from tables), and x = maximum pressure of water vapour *in inches* at the dew point. Then:—

(1) *Wet bulb reading above* 32° *F.*—

$$x = w - \frac{d}{88} \cdot \frac{h}{30}.$$

(2) *Wet bulb reading below* 32° *F.*—

$$x = w - \frac{d}{96} \cdot \frac{h}{30}.$$

Thus x is determined, and the dew point found from tables. All the rest follows. (See Example 4).

(*c*) Smithsonian Table.—This table is given in the Appendix, and the method of using it is indicated in Example 4. Centigrade temperatures are employed.

(*d*) By the Formula.—

$$F - f = \frac{B}{A \cdot S} \cdot P \cdot (t - t')$$

where B = specific heat of air, A = latent heat of steam, S = density of vapour compared with that of air, P = atmospheric pressure, and t, t' the two thermometer readings. F is taken from tables, the other factors on the right hand side are known or observed: thus f is found and all the rest follows.

Worked Examples.—(1) *Two cubic metres of moist air at* 17° *C. were drawn through a chemical hygrometer, and* 24·12 *grams of water were deposited in the tubes. Find the relative humidity of the air.*

$$\text{R.H.} = \frac{m}{m'}.$$

Here— m = 24·12 grams,

and m' denotes the mass of aqueous vapour necessary to saturate 2 cubic metres (*i.e.* 2,000 litres) at 17° C. The

maximum pressure of aqueous vapour at 17° C. = 14·4 mm. (See Table, p. 113.)

$$\therefore\ m' = \tfrac{5}{8}\left[2000 \times 1{\cdot}293 \times \frac{273}{290} \times \frac{14{\cdot}4}{760}\right]$$

$$= 28{\cdot}84 \text{ grams.}$$

$$\therefore\ \text{R.H} = \frac{m}{m'} = \frac{24{\cdot}12}{28{\cdot}84} = 0{\cdot}837 \text{ nearly};$$

or, the percentage humidity = 83·7.

(2) *Find the mass of* 1 *litre of moist air at* 15° *C., given that the dew point is* 10° *C., and the barometric height is* 759·13 *mm.*

The pressure of the aqueous vapour present = the maximum pressure of aqueous vapour at 10° C. = 9·13 mm., and the pressure of the air = 759·13 − 9·13 = 750 mm.

Hence :—

$$\text{The mass of the dry air} = \frac{1{\cdot}293 \times 750 \times 273}{760 \times 288}$$

$$= 1{\cdot}2095 \text{ grams};$$

$$\text{The mass of aqueous vapour} = \frac{{\cdot}808 \times 9{\cdot}13 \times 273}{760 \times 288}$$

$$= 0{\cdot}0092 \text{ grams.}$$

∴ total mass of one litre of moist air

$$= 1{\cdot}2095 + 0{\cdot}0092 = 1{\cdot}2187 \text{ grams.}$$

(3) *Find the hygrometric state of air at* 20° *C., the dew point being* 5° *C.*

The maximum pressure of aqueous vapour at—

$$5^\circ \text{ C.} = 6{\cdot}5 \text{ mm.} = f,$$

$$20^\circ \text{ C.} \doteq 17{\cdot}4 \text{ mm.} = \text{F}.$$

$$\therefore\ \text{R.H.} = \frac{f}{\text{F}} = \frac{6{\cdot}5}{17{\cdot}4} = {\cdot}374;$$

or, as a percentage— R.H. = 37·4.

(4) *Find the R.H. of the air when the dry bulb reads* 50° *F. and the wet bulb* 45° *F. Barometer reading is* 30 *inches.*

USING GLAISHER'S FACTORS :—

Factor corresponding to 50° F. = 2·1.

Difference in the readings = 5° F.

$\therefore$ Dew point (by Glaisher's Rule)

$$= 50 - (5 \times 2{\cdot}1) = 39{\cdot}5° \text{ F.}$$

Now 50° F. = 10° C. and 39·5° F. = 4·2° C.

Maximum pressure of vapour at 10° C. (from tables)

$$= 9{\cdot}165 \text{ mm.} = \text{F.}$$

Maximum pressure of vapour at 4·2° C. (from tables)

$$= 6{\cdot}2 \text{ mm.} = f.$$

$$\therefore \text{ R.H.} = \frac{f}{\text{F}} = \frac{6{\cdot}2}{9{\cdot}165} = 67{\cdot}6 \text{ per cent.}$$

USING APJOHN'S FORMULA :—

$d = 5°$ F. $h = 30$ inches.

$w =$ max. pressure of vapour at 45° F., *i.e.* at 7·2° C.

= 7·59 mm. (from tables)

= ·299 inch.

$$\text{Hence} \quad x = w - \frac{d}{88} \cdot \frac{h}{30}$$

$$= {\cdot}299 - \frac{5}{88} \cdot \frac{30}{30}$$

$$= {\cdot}242 \textit{ inch} = \text{max. press. at dew point.}$$

Further, max. press. of vapour at 50° F., *i.e.* at 10° C.

= 9·165 mm. (from tables) = ·360 *inch.*

$$\therefore \text{ R.H.} = \frac{f}{\text{F}} = \frac{{\cdot}242}{{\cdot}360} = 67{\cdot}1 \text{ per cent.}$$

If the dew point is required ·242 inch must be converted to mm. and the dew point found from tables.

USING SMITHSONIAN TABLE (Appendix).—The student should work the preceding by this table. Use Centigrade temperatures. From the top line look out the *difference* between the two thermometer readings. Go down this column and pick out the number that is on the same line as the dry bulb temperature in the first column. *This number is f.* Look down the column under O until the number f is reached: the temperature beside this is the dew point (if required). Look down the column under O, and pick out the number that has the air temperature beside it. *This number is F*, and R.H. = f/F. Interpolation is, of course, often necessary.

Exercises VIII.

(1) Find the mass of a litre of moist air at 18° C., the dew point being 5° C., and the barometric height 757·5 mm.

(2) The relative humidity of air at 16° C., expressed as a percentage, is 22·5; find the dew point.

(3) Two cubic metres of air, at 14° C., are found to contain 18·56 grams of moisture. Find the dew point and relative humidity of the air.

(4) Twenty litres of moist air, at 15° C., are drawn through a chemical hygrometer, and found to contain 0·1863 grams of moisture. What is the hygrometric state of the air?

(5) Two hundred c.cm. of hydrogen, measured at 15° C. and 754·68 mm. pressure, are collected over water. Find the mass of the hydrogen present. (1 litre of hydrogen at 0° C. and 760 mm. pressure weighs 0·0896 gram.)

(6) The dew point of air at 20° C. is 8° C. Find the relative humidity and the mass of aqueous vapour present in 1 litre of this air.

(7) Find the mass of dry air present in 10 litres of moist air, at 10° C. and 760 mm. pressure, the dew point of the air being 5° C.

(8) One hundred cubic centimetres of oxygen, saturated with water, are collected at a pressure of 740 mm. and a temperature of 15° C. Find the volume of dry oxygen at 0° C. and 760 mm., having given that the maximum pressure of aqueous vapour at 15° C. is 12·7 mm.

(9) A balloon contains 500 litres of hydrogen at 30° C. and 760 mm., which is saturated with aqueous vapour. The temperature of the external air is 20° C., its pressure 760 mm., and its relative humidity 50 per cent. Find the "ascensional force" of the balloon, given that the envelope weighs 200 grams. (Density of hydrogen relative to air is 0·0692).

(10) The following results were obtained with Regnault's hygrometer. Temperature of air = 16·5° C. Barometer = 758 mm. Dew appears 10·4° C. Dew disappears 10·6° C. Find the R.H. and the mass of 1 litre of the air.

(11) If the dry bulb reads 59° F. and the wet bulb 40° F., find the dew point and the R.H.

(12) Find the volume at 30° C. and pressure 760 mm. of 3 grams of moist air, whose hygrometric state is ·66 and coefficient of expansion ·0036.

CHAPTER IX.

CONDUCTION, CONVECTION, RADIATION.

Formulae for Conduction Calculations.—(1) *Absolute Conductivity*:—The important relation is :—

$$H = k \cdot \frac{A\theta t}{x} . \qquad \therefore k = \frac{Hx}{A\theta t} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where H = heat conducted across an area A through a thickness (or length) x in time t with a difference in temperature of θ degrees between opposite faces (or ends). k is the **absolute conductivity** of the material. Consistent units must be employed, the numerical value of k depending on the units of length, time, temperature and heat selected.

The following formula for H may be preferred :—

$$H = \frac{CADS}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where C = Conductivity, A = Area, D = temperature Difference, S = Seconds (or time) and T = Thickness (or length).

(2) *Diffusivity*:—If d = density, s = specific heat, k = conductivity, and κ = diffusivity, then :—

$$\kappa = \frac{\text{Conductivity}}{\text{Thermal capacity of unit volume}} = \frac{k}{s.d.} \quad \ldots\ldots (3)$$

(3) In the comparison of conductivities of rods by the wax melting experiment, *when the steady stage is reached*:—

$$\frac{\text{Conductivity of A}}{\text{Conductivity of B}} = \frac{(\text{Distance wax is melted along A})^2}{(\text{Distance wax is melted along B})^2} \quad \ldots\ldots (4)$$

Formulae for Radiation Calculations.—(1) The law of inverse squares applies to all radiation from a point source.

(2) *Jamin and Masson's Law*:—Let q = quantity of radiation entering a substance. After traversing unit thickness the quantity present is qa where a = a constant less than unity called the **coefficient of transmission.** On transmission through another unit thickness the quantity becomes a of qa, *i.e.* qa^2 and so on. Thus after transmission through a thickness n units we have:—

$$\text{Quantity of transmitted radiation} = qa^n \quad \ldots\ldots \ (5)$$

(3) *Newton's Law of Cooling*:—The rate of cooling is directly proportional to the excess of the temperature of the body over that of the enclosure. Thus:—

$$\text{Rate of cooling } \propto (\theta_1 - \theta_2) = \mu(\theta_1 - \theta_2) = \frac{\text{ES}}{ms}(\theta_1 - \theta_2) \quad \ldots \ (6)$$

where S = surface area of cooling body, m = its mass, s = its specific heat, θ_1 = temperature of cooling body, θ_2 = temperature of enclosure, and E = a constant called the **coefficient of emission.** If the body is below the enclosure in temperature a similar formula holds, the constant becoming the **coefficient of absorption.**

(4) *Dulong and Petit's Law of Cooling*:—For a given excess of temperature the rate of cooling increases in geometrical progression as the temperature of the enclosure increases in arithmetical progression. In formula form:—

$$\text{Rate of cooling} = \text{M}(a^{\theta_1} - a^{\theta_2}) \quad \ldots\ldots\ldots\ldots \ (7)$$

where M and a are constants, θ_1 = temp. of hot body, θ_2 = temp. of enclosure.

(5) *Stefan's Law*:—The radiating power of a body is proportional to the fourth power of its absolute temperature. The law of cooling then becomes:—

$$\text{Rate of cooling} = \text{M}(\text{T}_1^4 - \text{T}_2^4) \quad \ldots\ldots\ldots\ldots \ (8)$$

where T_1 = absolute temp. of body and T_2 = absolute temp. of enclosure. M is a constant.

(6) *Time taken by a body to cool*:—It is sometimes useful to have an expression for this. Using Newton's law it can be shown that the time taken to cool from one temperature θ_2 to another θ_1 is:—

$$\left.\begin{array}{r}\text{Time of}\\ \text{cooling}\end{array}\right\} = \frac{ms}{ES}\log_e\frac{\theta_2-\theta_0}{\theta_1-\theta_0} = 2\cdot3026\cdot\frac{ms}{ES}\cdot\log_{10}\frac{\theta_2-\theta_0}{\theta_1-\theta_0} \quad \dots (9)$$

θ_0 being the constant temperature of the enclosure: the other letters have their usual meaning.

Worked Examples. (1) *Find the quantity of heat that will be transmitted, in* 1 *hour, across a plate of copper* 1 *sq. metre in area and* 5 *cm. thick, the difference between the temperatures of its faces being* 10° *C.*

From the preceding we have:—

$$H = k\frac{A\theta t}{x}.$$

Adopting the C. G. S. system of units we have:—

$k = 1$ (look up table of conductivities),

$A = 1$ sq. metre $= 10000$ sq. cm. $\theta = 10°$ C.

$t = 1$ hour $= 3600$ seconds $x = 5$ cm.

$$\therefore H = \frac{10000 \times 10 \times 3600}{5}$$
$$= 72{,}000{,}000 \text{ calories.}$$

(2) *It is found that* 9162000 *gram-degrees of heat are transmitted, per minute, across a sheet of silver,* 100 *sq. cm. in area and* 1 *mm. thick, with a difference between the temperatures of its faces of* 100° *C. Find, in C.G.S. units, the absolute conductivity of silver.*

From the preceding we have:—

$$k = \frac{Hx}{A\theta t}.$$

Here:—$H = 9162000$ calories $A = 100$ sq. cm.

$x = \cdot1$ cm., $\theta = 100°$ C.

$t = 60$ seconds.

$$\therefore k = \frac{9162000 \times 0\cdot1}{100 \times 100 \times 60} = 1\cdot527 \text{ C.G.S. units.}$$

(3) *Sixty kilogram-degrees of heat are transmitted, in* 1 *minute, across a plate of copper,* 100 *sq. cm. in area and*

1 cm. thick and having 10° C. difference of temperature between its faces. Find the conductivity of copper in units involving the kilogram, metre, hour, and degree Centigrade.

Here, as in Example 2:—$k = \dfrac{Hx}{A\theta t}$;

and in the given units:—

H = 60 kilogram degrees A = 100 sq. cm. = ·01 sq. metre
x = 1 cm. = ·01 metre θ = 10° C.

$$t = 1 \text{ minute} = \tfrac{1}{60} \text{ hour.}$$

$$\therefore\ k = \frac{60 \times 0{\cdot}01 \times 60}{0{\cdot}01 \times 10} = 360 \text{ units.}$$

(4) *The conductivity of a certain material in the C.G.S. system is k: find its value in the British system, using the Centigrade scale, having given:—1 ft. = 30·48 cm., 1 sq. ft. = 929·01 sq. cm., 1 therm. (pound-degree-Centigrade unit)* = 453·59 calories.

In the British system to be used here, the conductivity is the number of therms flowing per square foot, per second, when the temperature gradient is 1° C. per foot.

The number of calories per second per square centimetre = k when the gradient is 1° C. per centimetre.

Therefore the number of therms per second per $\dfrac{1}{929{\cdot}01}$ sq. ft. $= \dfrac{k}{453{\cdot}59}$ when the gradient is 1° C. per centimetre.

Hence the number of therms per second per square foot $= \dfrac{929{\cdot}01k}{453{\cdot}59}$ when the gradient is 1° C. per centimetre, *i.e.* 1° C. per $\dfrac{1}{30{\cdot}48}$ ft.

But the rate of flow is directly proportional to the temperature gradient: hence we finally get the following:—

The number of therms per second per square foot $= \dfrac{929{\cdot}01k}{453{\cdot}59 \times 30{\cdot}48} = {\cdot}0672k$ when the gradient is 1° C. per foot and this is the conductivity required.

(5) *The specific heat of copper is 0·095, and its density is 8·9; find, in C.G.S. units, the measure of its diffusivity. Find the thickness of a plate of copper that would be raised in temperature through 1° C. by the heat transmitted, in unit time, through another copper plate of the same area and 1 cm. in thickness, with a difference of temperature of 1° C. between its faces. Also find the number of degrees rise of temperature produced in a plate of copper of the same area and 1 cm. in thickness, by the same flow of heat.*

By (3):— $\kappa = \dfrac{k}{sd}.$

Here— $k = 1, \; s = 0{\cdot}095, \; d = 8{\cdot}9.$

$$\therefore \; \kappa = \frac{1}{0{\cdot}095 \times 8{\cdot}9} = \frac{1}{{\cdot}8455} = 1{\cdot}183.$$

Let A sq. cm. be the area of the plates: the heat transmitted is:—

$$\mathrm{H} = k\frac{A\theta t}{x} = 1\,\frac{\mathrm{A} \times 1 \times 1}{1} = \mathrm{A} \text{ gram-degrees.}$$

If y denote the thickness of the copper plate which will be raised 1° C. by this quantity, A, of heat, then:—

$$\mathrm{A} = m.s.1 = \mathrm{A}yd.s.1 = \mathrm{A}yds.$$

$$\therefore \; 1 = yds. \quad i.e.\; y = \frac{1}{sd} = 1{\cdot}183 \text{ cm.}$$

Hence the measure of the thickness is also the measure of the diffusivity. Further, let n denote the number of degrees rise produced in a plate of area A and 1 cm. thick, by this quantity, A, of heat, then, as above—

$$\mathrm{A} = msn = \mathrm{A}.1.dsn = \mathrm{A}dsn.$$

$$\therefore \; 1 = dsn. \quad \therefore \; n = \frac{1}{sd} = 1{\cdot}183^\circ \text{ C.}$$

Thus diffusivity is also measured by this rise of temperature; hence the term **thermometric conductivity.**

(6) *A piece of rock salt 2·5 mm. thick allows 95°/₀ of the incident radiant heat to pass through it. How much will a piece four times as thick transmit? What is the coefficient of transmission of rock salt?*

95% gets through the first plate.

95% of 95% gets through a plate twice as thick.

95% of 95% of 95% gets through a plate three times as thick.

95% of 95% of 95% of 95% gets through a plate four times as thick.

i.e. 81·45 % gets through a plate four times as thick.

Further, this plate *will have unit thickness, i.e.* 1 *cm.*

∴ By the definition, the coefficient of transmission = ·8145.

(7) *An iron ball 2 cm. diameter is heated and fixed in an enclosure coated with lampblack on the inside. The space is to be regarded as a vacuum. How long will it take the ball to cool from 40° C. to 20° C. assuming the enclosure to be at a constant temperature 10° C. Density of iron = 8 grams per c.cm., specific heat* = ·112, *coefficient of emission for iron* = ·000017.

$$\text{Vol. of ball} = \tfrac{4}{3} \times \pi \times r^3 = \tfrac{4}{3} \times \tfrac{22}{7} \times 1 = \tfrac{88}{21} \text{ c.cm.}$$

$$\therefore m = \text{mass of iron} = \tfrac{88}{21} \times 8 = 33{\cdot}5 \text{ grms.}$$

Further, S = surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times 1 = 12{\cdot}5$ sq. cm.

Hence :—

$$\text{Time taken to cool} = \frac{ms}{ES} \log \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} \times 2{\cdot}3026$$

$$= \frac{33{\cdot}5 \times {\cdot}112}{{\cdot}000017 \times 12{\cdot}5} \times \log \frac{40 - 10}{20 - 10} \times 2{\cdot}3026$$

$$= 17656{\cdot}5 \times \log 3 \times 2{\cdot}3026$$

$$= 17656{\cdot}5 \times {\cdot}47712 \times 2{\cdot}3026$$

$= 19422$ seconds approx. *or* about 5h. 24 m.

(8) *Using Stefan's law, compare the heat lost per second by an iron ball at 427° C. in an evacuated vessel at 27° C. with the heat lost by the same ball at 227° C. in the same vessel at 27° C.*

The absolute temperatures are 700, 500 and 300.

Case 1—Rate of cooling = M ($700^4 - 300^4$).

Case 2— „ „ = M ($500^4 - 300^4$).

$$\therefore \frac{\text{Heat lost per sec. in Case 1}}{\text{Heat lost per sec. in Case 2}} = \frac{700^4 - 300^4}{500^4 - 300^4}$$

$$= \frac{400 \times 1000 \times (700^2 + 300^2)}{200 \times 800 \times (500^2 + 300^2)} = 42 \cdot 6.$$

(9) *Find the time required to form a thickness of 2 cm. of ice on a pond, when the air in contact with its upper surface is maintained at* -12° *C. (conductivity of ice* $= \cdot 0022$ *in C.G.S. units, density of ice* $= \cdot 917$*).*

As the sheet of ice forms on the pond the latent heat of liquefaction is conducted away through the ice.

Let l cm. be the thickness of ice at any given instant, and let a further small thickness dl freeze during the next small interval of time dt. Then:—

H = heat emitted by 1 sq. cm. of the surface
= mass of ice formed × latent heat of liquefaction
= volume of ice formed × density of ice × latent heat
$= dl \times 1 \times \cdot 917 \times 80.$

$\therefore$ H $= 73 \cdot 36\, dl$ units.

Since this heat is conducted away through a thickness of l cm. of ice we have, by the usual formula:—

$$H = k \cdot \frac{A\theta t}{x}, \quad \text{so that} \quad 73 \cdot 36\, dl = k \cdot \frac{1 \times T \times dt}{l},$$

where T is the difference in temperature of the upper and lower surfaces of the ice; hence:—

$$\frac{kT}{73 \cdot 36} \cdot dt = l\, dl,$$

the integration of which gives:—

$$\frac{kT}{73 \cdot 36} \cdot t = \frac{l^2}{2}, \quad \text{or} \quad t = 36 \cdot 68 \frac{l^2}{kT}.$$

In the present case $k = \cdot 0022$, $T = 12$, and $l = 2$.

$$\therefore t = \frac{36 \cdot 68 \times 4}{\cdot 0022 \times 12} \text{ seconds} = 92 \cdot 6 \text{ minutes}.$$

Exercises IX.

(1) Peclet has stated that the quantity of heat which passes, in an hour, through a plate of lead 1 sq. metre in area and 1 cm. thick, with a difference of 1° C. between the temperatures of its surfaces, is 1383 Kilogram-degrees. What value does this give for the absolute conductivity of lead in the C.G.S. system?

(2) The absolute conductivity of copper in C.G.S. units is 1; how many heat-units will pass, per minute, across a plate of copper, 1 metre long, 1 metre broad, and 5 cm. thick, when its opposite faces are kept at temperatures differing by 100° C.?

(3) The thermal conductivity of felt, in C.G.S. units, is 0·000087; find the quantity of heat that is transmitted, in one hour, through a layer of felt 1 cm. in thickness and 20 sq. cm. in area, when its opposite faces are kept at temperatures differing by 20° C.

(4) Calculate the quantity of heat lost, per hour, from each square metre of the surface of an iron steam boiler 0·8 cm. in thickness, when the temperature of the inner surface of the boiler is 120° C. and that of the outer surface 119·5° C., the coefficient of conductivity of iron being 11·5 (referred to 1 cm. as unit of length, 1 minute as unit of time, and the quantity of heat required to raise the temperature of one gram of water from 0° to 1° C. as unit of heat).

(5) Express the conductivity of copper in units involving the pound, foot, second, and degree Fahrenheit.

(6) A square metre of a substance, 1 cm. thick, has one side kept at 100° C., and the other, by means of ice, at 0° C. In the course of 10 minutes one kilogram of ice is melted by this operation. Calculate the conductivity of the substance, assuming the latent heat of water to be 80.

(7) The mean temperature of the earth at a depth of 972 feet being 23° C., and 14° C. at the surface, and the average estimated loss of heat per square foot of surface in 27 years being 4·5 units of heat, find the coefficient of conductivity per cubic foot per hundred years.

(8) Estimate the rate at which ice will form on a pond when it is 5 cm. thick and the temperature of the air is -12 °C., the conductivity of ice being ·0022.

---

(9) If the thermal conductivity of ice be ·005 find the rate at which the ice on a pond will increase in thickness if the air be at -20° C. Plot a curve showing how the rate varies as the thickness of the ice increases.

(10) If a certain piece of coloured glass 1 mm. thick transmits half the radiation that passes into it from a sodium flame, how much would be transmitted by a plate nine times as thick? What is the coefficient of transmission of this particular glass?

(11) Find how many calories are radiated in $\frac{1}{4}$ hour from the surface of a solid copper cube of 10 cm. edge *maintained* at 20° C. in an enclosure surrounded by melting ice at 0° C. (coefficient of emission of copper = ·00002 in C.G.S. units). Use Newton's law.

If this cube were allowed to cool, find how long it would take it to cool from 20° C. to 10° C. (specific heat of copper = ·095, density of copper = 8·7 grams per c.cm.).

(12) A copper ball of diameter 2 cm. is suspended at 100° C. in an enclosure, the walls of which are at 0° C. After 4 minutes the temperature of the ball has fallen to 64° C. Find the coefficient of emission of copper given that its specific heat is ·095 and its density 8·8 grams per c.cm. {Use formula (9) *or* use (6) taking $\theta_1 = \frac{1}{2}(100 + 64)$.}

CHAPTER X.

THE MECHANICAL EQUIVALENT OF HEAT: INTERNAL AND EXTERNAL WORK: POROUS PLUG EXPERIMENT: EFFICIENCY.

Formulae for Calculations.—(1) *The first law of Thermo-dynamics.*—If **W** denote a definite quantity of work, **H** the equivalent quantity of heat, and **J** the mechanical equivalent of heat, then :—

$$\mathbf{W} = \mathbf{JH} \quad \ldots\ldots \quad (1)$$

expresses the first law of thermo-dynamics.

(2) *Values of J.*—The values of J, which will often be required, are as follows :—

$$
\begin{aligned}
\mathbf{J} &= 778 \text{ (pound, foot, degree Fahrenheit)} \\
&= 1400 \text{ (,, ,, ,, Centigrade)} \\
&= 427 \text{ (gram, metre, ,, ,,)} \\
&= 42700 \text{ (gram, cm., ,, ,,)} \\
&= 4 \cdot 19 \times 10^7 \text{ (ergs per calorie)} \\
&= 4 \cdot 19 \text{ (joules per calorie)}
\end{aligned}
$$

(3) *External work done by a gas during expansion at constant pressure.*—As already indicated in Chapter IV, this is given by :—

$$w = pv \quad \ldots\ldots \quad (2)$$

where p = pressure per unit area, and v = *increase in volume.* It is also shown that for one degree rise in temperature at constant pressure :—

$$\mathrm{W} = \frac{\mathrm{PV}}{\mathrm{T}} = \mathrm{R} \quad \ldots\ldots \quad (3)$$

where **P** = pressure, **V** = volume, and **T** = absolute temperature of the gas.

(4) *Determination of J from the difference between the two specific heats of a gas.*—Consider *unit mass* of gas. Let s_p = specific heat at constant pressure, s_v = specific heat at constant volume: then:—

$s_p - s_v$ = heat equivalent to the external work done when unit mass of the gas is raised one degree in temperature at constant pressure.

$\therefore\ s_p - s_v = \dfrac{W}{J}$, where W = the external work in question:

$$\therefore\ J = \frac{W}{s_p - s_v} \qquad \text{........ ....} \quad (4)$$

(5) *Internal and external latent heats.*—The important case is that of vaporisation of water, and the method of dealing with such problems will best be gathered from the following:—

Consider 1 pound of water in a cylinder closed by a piston. Let the area of the piston be 1 sq. ft. and the pressure p pounds per sq. inch. Imagine the water to be at the boiling temperature for this pressure. The volume of the water may be taken to be ·017 c.ft. Apply heat until all is turned to steam. The heat supplied will be the latent heat L. Let the volume now be V c.ft. The increase in volume is (V − ·017) c.ft. The pressure is $144p$ pounds per sq. ft. The external work done is, therefore, $144p\,(V - \cdot 017)$ foot pounds, and the heat equivalent to this is $144p\,(V - \cdot 017)/J$, where J is in the appropriate units (foot pounds): thus:—

$$\text{External latent heat} = \frac{144p\,(V - \cdot 017)}{J} \text{ heat units} \quad \text{..............} \quad (5)$$

$$\text{Internal latent heat} = \left\{ L - \frac{144p\,(V - \cdot 017)}{J} \right\} \text{ heat units} \quad \text{......} \quad (6)$$

Note that 1 c.cm. of water at 100° C. and at normal pressure becomes about 1,670 c.cm. of steam at 100° C., that the latent heat of steam is about 967 B.Th.U. (pound degree Fahrenheit), or 540 calories (also pound degree Centigrade), and that, in the latter case, the external latent heat is about 41 and the internal latent heat 499.

(6) *Saturated steam details for practical problems.*—In calculations it is often necessary to know corresponding values of the pressure and temperature of saturated steam.

These are taken from tables in practice (see Appendix), but in the absence of tables the following formula may be employed:—

$$\text{Rankine:—}\log_{10} p = 6{\cdot}1007 - \frac{B}{T} - \frac{C}{T^2},$$

where p = pressure in lb. per sq. inch, T = absolute temperature (Centigrade), $\log_{10}B = 3{\cdot}1812$ and $\log_{10}C = 5{\cdot}0881$.

Similarly, corresponding values of pressure and volume are taken from the tables, but many formulae have been devised, *e.g.*

$$\text{Rankine:—}PV^{\frac{17}{16}} = 479,$$

$$\text{Zeuner:—}PV^{1{\cdot}0646} = 479,$$

where P = pressure in lb. per sq. inch, and V = volume in c.ft. of 1 lb. of steam.

The formulae for the total heat and latent heat of steam at different temperatures given in Chapter VII. must be well known. The *specific heat of superheated steam* is about ·48.

(7) *The "Porous Plug" experiment.*—A frequent problem is that of a liquid being forced through a porous plug and the calculation of J from the data of the problem. In this case we have:—

$$s = \frac{p'V}{t'}(1 - aT) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where s = specific heat *in mechanical units*, p' = difference in pressure on the two sides of the plug, T = the absolute temperature of the liquid, t' = change in temperature in passing through, V = volume of unit mass of liquid, and a = coefficient of expansion. Thus, knowing s in heat units and also in mechanical units, the value of J is determined. (See Example 7, p. 79.)

Another type of "porous plug" problem is that of a gas being forced through the plug, and the data may be used to express on the absolute thermodynamic scale any temperature given by a gas thermometer. The most convenient formula is:—

$$T = t + \frac{1}{a_0} - \frac{{\cdot}4343 s_p t'}{P_0 V_0 a_0 (\log P - \log p)} \quad \ldots\ldots\ldots \quad (8)$$

where T = absolute temperature on the thermodynamic scale corresponding to t on the gas thermometer scale, P and p = the pressures on the two sides, and α_0 = the expansion coefficient. The other letters have their usual meaning. s_p is in *mechanical* units. V_0 is the volume of unit mass. (See Example 8, p. 79.)

(8) *Efficiency of an engine.*—If H = heat received from the source, and h = heat converted into work:—

$$\text{Efficiency} = \frac{h}{H} \quad \ldots\ldots\ \ldots\ldots\ldots\ldots \quad (9)$$

Note that 1 horse-power = 33,000 ft. pounds per minute.

Worked Examples.—(1) *A mass of* 10 *pounds falls to the ground from a height of* 695 *feet. Assuming that it does not rebound, find the heat liberated by its impact on the ground.*

Here, work done = 6954 foot-pounds. Taking $J = 1400$, the heat equivalent of this work is $= \frac{6950}{1400} = 5$ pound-degrees (Centigrade) approx.

(2) *With what velocity must a lead bullet at* 50° *C. strike against an obstacle in order that the heat produced by the arrest of its motion, if all produced within the bullet, might be just sufficient to melt it? Specific heat of lead* = ·031. *Melting point of lead* = 335° *C. Latent heat of fusion of lead* = 5·37. *Mechanical equivalent of heat* = 1390 ft.-lb.

Let m lb. = mass of bullet, v ft. per sec. = its velocity: then:—

$$\text{Kinetic energy of bullet} = \tfrac{1}{2}mv^2 \text{ foot-poundals.}$$

$$= \frac{mv^2}{64} \text{ foot-pounds.}$$

$$\text{Heat equivalent to this} = \frac{mv^2}{64 \times 1390} \text{ units.}$$

$$\text{Heat required to raise bullet to melting point} = mst$$
$$= m \times \cdot 031 \times 285.$$

$$\text{Heat required to melt bullet} = mL = m \times 5\cdot 37$$

$$\therefore \frac{mv^2}{64 \times 1390} = (m \times \cdot 031 \times 285) + (m \times 5\cdot 37),$$

i.e. $\frac{v^2}{64 \times 1390} = (\cdot 031 \times 285) + 5\cdot 37$ *or* $v^2 = 211676\cdot 8$.

$$\therefore\ v \doteq 460\cdot 08 \text{ feet per second.}$$

(3) *Taking the mechanical equivalent as* 1,400 *ft.-lb., find the heat produced in stopping by friction a fly-wheel,* 112 *lb. in mass and* 2 *ft. in radius, rotating at the rate of one turn per second, assuming the whole mass to be concentrated in the rim.*

$$\text{Kinetic energy of wheel} = \frac{mv^2}{64} \text{ ft.-lb.}$$

$$= \frac{112 \times 16 \times \frac{22}{7} \times \frac{22}{7}}{64} \text{ ft. lb.}$$

$$\text{Heat equivalent to this} = \frac{112 \times 16 \times 22 \times 22}{7 \times 7 \times 64 \times 1400} = \frac{9\cdot 68}{4900}$$

$$= \cdot 19 \text{ unit (pound-degree-C.)}$$

(4) *A cylindrical calorimeter is suspended by a single wire, so that it is capable of rotation about a vertical axis. A paddle wheel rotates in water inside at a speed of* 1,200 *revolutions per minute, and the calorimeter is just prevented from rotating by having two strings fixed to the circumference at opposite ends of a diameter, the strings then passing over two pulleys, and each carrying a weight of* 300 *grams. The diameter of the calorimeter is* 30 *cm. Find the heat developed in the calorimeter per second.*

$$\text{Moment of couple} = 300 \times 981 \times 30 \text{ dyne cm.}$$

$$\therefore \text{ Work per revolution} = 300 \times 981 \times 30 \times 2\pi \text{ ergs.}$$

$$\therefore \text{ Work per second} = 300 \times 981 \times 30 \times 2\pi \times \tfrac{1200}{60} \text{ ergs.}$$

$$\text{Hence :—Heat, per second} = \frac{\text{Work per second}}{J}$$

$$= \frac{300 \times 981 \times 30 \times 20 \times 2 \times 3\cdot 1416}{4\cdot 2 \times 10^7}$$

$$= 26\cdot 42 \text{ calories.}$$

(5) *If the specific heat of air at constant pressure be taken as ·2375 calorie, and the specific heat at constant volume as ·1685 calorie, determine the value of J in ergs per calorie.*

$$J = \frac{w}{s_p - s_v} = \frac{w}{\cdot 069}.$$

Now:—1 c.cm. of air at 0° C. and 76 cm. pressure
$= \cdot 001293$ grm.

$\therefore$ 1 gram at 0° C. and 76 cm. $= 773{\cdot}3$ c.cm. approx.

And:—*Increase* in volume when raised 1° C. $= \dfrac{773{\cdot}3}{273}$ c.cm.

Pressure = that due to 76 cm. mercury $= 1033{\cdot}3$ grm. weight.

$$\therefore\ w = Pv = 1033{\cdot}3 \times \frac{773{\cdot}3}{273} \text{ grm. centimetres.}$$
$$= 2871300 \text{ ergs.}$$

Hence:— $J = 2871300 \div \cdot 069.$
$= 4{\cdot}16 \times 10^7$ ergs per calorie.

(6) *Three pounds of water are supplied to a boiler at 55° F. and evaporated at 309° F. (Volume per lb. = 5·6 c.ft.) Find the sensible heat, the total latent heat, and the external and internal latent heats.*

(1) Sensible heat = heat used in raising the temp. of 3 lb. of water from 55° F. to 309° F.
$= 3\,(309 - 55) = 762$ B.Th.U. (pound-degree-Fahrenheit).

(2) Total latent heat of the three pounds
$= 3\,[1114 - \cdot 7t°\text{ F.}]$ approx. $= 3\,(1114 - \cdot 7 \times 309)$
$= 2693$ B.Th.U.

(3) External latent heat of the three pounds:—
$= 3\left[\dfrac{144p\,(V - \cdot 017)}{J}\right]$, and since from tables, p for saturated steam at 309° F. $= 76{\cdot}7$ lb. per sq. inch

$$= \frac{3 \times 144 \times 76{\cdot}7 \times (5{\cdot}6 - \cdot 017)}{778} = 238 \text{ B.Th.U.}$$

(4) Internal latent heat of the three pounds

$$= 2693 - 238 = 2455 \text{ B.Th.U.}$$

(7) *A very long cylinder* 1 *sq. cm. in cross-section is fixed vertically. It is open at the top, but closed at the bottom by a plug with very small holes in it. It contains mercury at* 0° *C. standing* 10 *metres high. The mercury rises* ·7° *C. in temperature in passing through the holes. Find the mechanical equivalent of heat.* (*Specific heat of mercury* = ·032, *coefficient of expansion* = ·0002.)

By (7) :— $$s = \frac{p'V}{t'}(1 - aT).$$

Here, therefore :—

$$(J \times \cdot 032) = \frac{(1000 \times 13{\cdot}6 \times 981) \times \frac{1}{13{\cdot}6} \times (1 - \cdot 0002 \times 273)}{\cdot 7}$$

$$\therefore\ J = 4{\cdot}1 \times 10^7 \text{ ergs per calorie.}$$

Note that s is in mechanical units $(0{\cdot}032 \times J)$, p' is in dynes per sq. cm., V is the volume of *unit* mass $\left(\frac{1}{13{\cdot}6}\text{ c.cm.}\right)$, and T is the absolute temperature corresponding to 0° C. (273).

(8) *One gram of air at* 0° *C. and under a pressure of* 1 *megadyne per sq. cm. has a volume of* 783 *c.cm. Air at* 0° *C. is forced through in the porous plug experiment, the pressure on one side being* 3·689 *atmospheres, and on the other* 1·004 *atmospheres. The temperature falls* ·6761°. *Find the absolute zero.* (*Coefficient of expansion* = $\frac{1}{272}$.)

$$T = t + \frac{1}{a_0} - \frac{\cdot 4343 s_p t'}{P_0 V_0 a_0 (\log P - \log p)}.$$

$$T = 0 + \frac{1}{\frac{1}{272}} - \frac{\cdot 4343 \times (\cdot 24 \times 4{\cdot}2 \times 10^7) \times (-\cdot 6761)}{10^6 \times 783 \times \frac{1}{272} \times (\log 3{\cdot}689 - \log 1{\cdot}004)}$$

$$= 272 + \frac{\cdot 4343 \times (\cdot 24 \times 4{\cdot}2 \times 10^7) \times \cdot 6761 \times 272}{10^6 \times 783 \times \cdot 54989}$$

$$= 272 + 1{\cdot}87$$

$$= 273{\cdot}87. \quad \therefore \text{ Abs. zero} = -273{\cdot}87°.$$

(9) *An engine consumes* 3 *pounds of coal per horse-power per hour. The heat developed by the combustion of* 1 *pound of coal is capable of converting* 15 *pounds of water at* 100° *C. into steam at* 100° *C. Find the efficiency of the engine.* (J = 1390).

1 lb. of coal produces 15×537 *pound-degree-C. units* of heat.

$\therefore$ 3 lb. „ „ produce $3 \times 15 \times 537$ „ „ „ „ „ „

That is, the heat absorbed by the engine per hour is

$$3 \times 15 \times 537 \text{ pound-degree-C. units.}$$

And the work performed by the engine per hour is equivalent to—

$$\frac{33000 \times 60}{1390} \text{ pound-degree-C. units.}$$

Therefore, the efficiency of the engine is given by—

$$e = \frac{h}{\text{H}} = \frac{33000 \times 60}{1390 \times 3 \times 15 \times 537} = 0{\cdot}0589 = 5{\cdot}89 \text{ per cent.}$$

(10) *An engine gives one horse-power for an hour for each* 2 *lb. of coal consumed. The heating value of the coal (best Welsh) is* 16,000 *B.Th.U. per lb.* **What percentage of the heat in the coal is spent in mechanical work?**

The feed water enters the boiler of the above plant at 252° *F., and for each pound of coal consumed* 10 *lb. of water are turned to steam, the steam pressure being* 175 *lb. per sq. inch.* **What percentage of the heat in the coal is taken up by the steam leaving the boiler?**

The engine gives one horse-power for an hour for every 20 *lb. of steam supplied to it. The steam is exhausted and condensed at* 140° *F.* **What percentage of the energy of the steam is spent in mechanical work?**

(*a*) H = Heat supplied to engine per hour

$$= 16000 \times 2 = 32000 \text{ B.Th.U.}$$

Work done by engine per hour

$$= 33000 \times 60 = 1980000 \text{ ft.-pounds.}$$

h = Heat equivalent to this = $\frac{1980000}{778}$ B.Th.U.

$$\therefore \text{ Percentage required} = \frac{h}{\text{H}} \times 100 = \frac{1980000 \times 100}{778 \times 32000}$$

$$= 8{\cdot}0.$$

(*b*) From tables, the total heat of steam at 175 pounds pressure is 1195 B.Th.U. (approx.). The sensible heat of water at 252° F. is 252 — 32, *i.e.* 220 B.Th.U. Hence every pound of steam in the boiler has been given 1195 — 220, *i.e.* 975 B.Th.U. Thus 10 lb. have received 975 × 10, *i.e.* 9750 B.Th.U., and the heating value of the 1 lb. of coal consumed is 16000 B.Th.U.

$$\therefore \text{ Percentage required} = \tfrac{9750}{16000} \times 100$$
$$= 61.$$

(*c*) Total heat of 1 lb. of steam entering the engine

$$= 1195 \text{ B.Th.U.}$$

Heat in 1 lb. of water at the condenser at 140° F.

$$= 140 - 32 = 108 \text{ B.Th.U.}$$

$\therefore$ Heat available per 1 lb. of steam

$$= 1195 - 108 = 1087 \text{ B.Th.U.,}$$

and heat available per 20 lb. of steam

$$= 1087 \times 20 = 21740 \text{ B.Th.U.}$$

Now, work done by engine per hour

$$= 33000 \times 60 \times 1980000 \text{ ft.-pounds,}$$

and heat equivalent to this $= \tfrac{1980000}{778}$.

$$\therefore \text{ Percentage required} = \frac{1980000 \times 100}{778 \times 21740}$$
$$= 11{\cdot}7.$$

Clearly the boiler is the most efficient item of the plant.

The Three Fundamental Equations of the Kinetic Theory of Gases.—For reference, the various forms are summarised below :—

First Fundamental Equation $\begin{cases} \text{M.K.E. per mass } m = \frac{1}{2}mu^2. \\ \text{M.K.E. per unit volume} = \frac{1}{2}\rho u^2. \\ \text{Mean M.K.E. per molecule} = \frac{1}{2}\mu u^2. \end{cases}$

Second Fundamental Equation $\begin{cases} p = \frac{1}{3}\rho u^2 \;\; or \;\; pv = \frac{1}{3}u^2. \\ p = \frac{2}{3}\text{M.K.E. per unit volume.} \end{cases}$

Third Fundamental Equation $\begin{cases} u^2 = 3\text{RT}. \\ \text{M.K.E. per unit mass} = \frac{3}{2}\text{RT}. \\ \text{M.K.E. per grm. molecule} = \frac{3}{2}\overline{\text{R}}\text{T}. \end{cases}$

where M.K.E. = molecular kinetic energy, u = velocity of mean square of the molecules, ρ = absolute density, μ =

molecular mass, T = absolute temperature, R = gas constant (dealing with unit mass), and $\bar{R}$ = absolute gas constant (dealing with the gram-molecule).

Worked Example.—(11) *Find the M.K.E. per gram-molecule of any gas at 27° C. Find also the M.K.E. of 1 grm. of oxygen at 27° C. and the velocity of mean square of its molecules* (*Jude*).

Since $\bar{R}$ referred to megadynes per sq. cm. = 83 approximately (prove this) the M.K.E. per grm.-molecule = $\frac{3}{2}\bar{R}T = \frac{3}{2} \times 83 \times (273 + 27) = 37{,}350$ megalergs.

Molecular mass of oxygen = 32, $\therefore$ its grm.-molecule = 32 grms. Hence M.K.E. per grm. of oxygen = 37,350/32 = 1167 megalergs.

Again, M.K.E. *per grm.* $= \frac{1}{2}u^2$, $\therefore$ $\frac{1}{2}u^2 = 1167$, *i.e.* $u^2 = 2334$, or $u = 48$ decametres per sec. = 48,000 cm. per second.

Exercises X.

(1) Assuming that the mass of a cubic foot of steam at 100° C. and 760 mm. pressure is 240 grains, find what fraction of the latent heat of steam is consumed in doing external work, *i.e.* in lifting the atmosphere.

(2) How can the amount of work done against external pressure during change of volume be expressed numerically?

1 gram of air is heated under constant pressure from 0° to 10° C.; determine the work, either in *ergs* or in *centimetre-grams*, due to the expansion.

(3) Distinguish between the specific heat of air under constant pressure and its specific heat at constant volume. Show how, from a knowledge of the two specific heats of air, together with its density at given pressure and temperature, the value of the mechanical equivalent of heat may be computed.

(4) Given that the ratio of the two specific heats of air is 1·41, and that the work done during expansion, at normal pressure, by 1 gram of air when its temperature is raised from 0° C. to 1° C. is 2926 centimetre-grams, find the value of the two specific heats.

(5) An engine consumes 40 pounds of coal of such calorific power that the heat developed by the combustion of 1 pound is capable of converting 16 pounds of water at 100° C. into steam at the same tem-

perature, and during the process the engine performs 16,000,000 foot-pounds of work. What percentage of the heat produced is wasted?

(6) What is the efficiency of an engine which consumes 28 pounds of coal in drawing a train one mile against a resistance equal to the weight of $1\frac{1}{2}$ tons, the calorific power of the coal being such that 1 pound is capable of converting 16 pounds of boiling water into steam at the same temperature?

(7) What heat must be given to 1 lb. of water at 80° F. to convert it into steam at 300° F.? How many pounds of this steam are equivalent in *total heat* to the calorific power of a pound of best Welsh coal?

(8) In a steam boiler the steam pressure was 150 lb. per square inch absolute, and the temperature of the feed supply 120° F. The heating value of the coal was 15,200 B.Th.U. per lb., and 11 lb. of water were evaporated per lb. of coal burned. What percentage of the heat of the coal was taken up by the steam leaving the boiler. The steam is assumed dry.

(9) An engine gives 1 H.P. (indicated) for a consumption of 15 lb. weight per hour of dry steam at a pressure of 195 lb. per sq. inch absolute in the steam chest. How much heat would be supplied to the engine per I.H.P. per hour?

(10) How much heat per I.H.P. per hour is available for conversion into work in the case of the engine in Question 9, given that condensation is at 130° F.? What percentage of this is really converted into work?

(11) The volume of 1 gram of air at 0° C. and a pressure of 10^6 dynes per sq. cm. is 783·8 c.c.; at 2×10^6 dynes it is 391·0 c.c. Air expanding through a porous plug from 2×10^6 to 10^6 dynes pressure is cooled ·25° C. The specific heat of air at constant volume is ·17. How much of the observed cooling is due to work done in the change of volume? How much would air at a pressure of 2×10^6 dynes be cooled if it expanded to double its volume without doing external work?

(12) Air at 16° C. is forced through a porous plug, the pressure on the two sides of the plug being 20·943 lb. per sq. inch and atmospheric pressure (14·777 lb. per sq. inch) respectively. The air is cooled 0·105° C. in passing through the plug. Determine the absolute temperature on the thermodynamic scale corresponding to the initial temperature of 16° C.

Specific heat of air at constant pressure = 0·2375.

Coefficient of expansion of air = 0·003665.

CHAPTER XI.

ISOTHERMAL AND ADIABATIC CHANGES: CARNOT'S PERFECT ENGINE AND APPLICATIONS OF CARNOT'S PRINCIPLE: ENTROPY.

Formulae for Calculations.—(1) *Isothermal Changes.* The relation between P and V during these changes of a perfect gas is :—

$$PV = \text{a constant } \textit{or } P_1V_1 = P_2V_2 \quad\ldots\ldots\ldots\ldots (1)$$

and the work done during, say, an isothermal *compression* (work done **on** the gas) is :—

$$W = P_1V_1 \text{ (or } P_2V_2\text{)}\log_e \frac{V_1}{V_2}$$

$$= P_1V_1 \text{ (or } P_2V_2\text{)}\log_{10} \frac{V_1}{V_2} \times 2{\cdot}3026 \quad\ldots\ldots\ldots (2)$$

where P_1,V_1 = initial and P_2,V_2 = final conditions. For *expansion* (work done **by** the gas) we have $W = P_1V_1$ (or $P_2V_2)\log_e \frac{V_2}{V_1}$.

(2) *Adiabatic Changes.*—In the following, γ = ratio of the specific heat at constant pressure to the specific heat at constant volume and T = absolute temperature.

(*a*) Relation between P and V :—

$$PV^\gamma = \text{a constant } \textit{or } \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right) \quad\ldots\ldots\ldots\ldots (3)$$

(*b*) Relation between V and T :—

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \textit{ or } \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad\ldots\ldots\ldots (4)$$

(*c*) Relation between P and T :—

$$\left(\frac{P_2}{P_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^{\gamma} \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

and the work done during, say, an adiabatic *compression* is :—

$$W = \frac{P_2V_2 - P_1V_1}{\gamma - 1} \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

while for *expansion* we have $W = (P_1V_1 - P_2V_2)/(\gamma - 1)$.

(3) *Adiabatics of a Saturated Vapour.*—In the case of a *saturated* vapour undergoing an adiabatic transformation :—

$$\frac{xL}{T} + s \log_e T = \text{a constant} \quad\ldots\ldots\ldots \quad (7)$$

where $x =$ dryness fraction (Chapter VII.), $L =$ latent heat of the vapour, $s =$ specific heat of the liquid, and $T =$ absolute temperature.

(4) *Elasticities of a Gas.*—The isothermal elasticity of a gas is equal to the pressure : the adiabatic elasticity is equal to γ times the pressure.

(5) *Efficiency of a Perfect Heat Engine.*—The efficiency of Carnot's perfect heat engine is given by :—

$$\text{Efficiency} = \frac{H - h}{H} = \frac{S - T}{S} \quad\ldots\ldots\ldots\ldots \quad (8)$$

where $H =$ heat taken from the *source*, $h =$ heat returned to the *refrigerator*, $S =$ absolute temperature of the source, and $T =$ absolute temperature of the refrigerator ($\therefore H - h$ $=$ heat converted into work). Strictly *thermodynamic* temperatures are meant, but it is not usual to be so precise, and in calculations the absolute temperatures in the elementary sense are used ($273 + t^\circ$ C.).

Note that $(H - h)/H$ is the *thermal* efficiency of *any* engine (steam, gas, etc.) where $H =$ heat taken from the "hot" body (the "boiler"), and $h =$ heat rejected as waste to the "cold" body (the "condenser" or "exhaust") as is indicated in Chapter X., but the value of this for an actual engine will be less than for Carnot's engine working between the same two temperatures. Further this "*thermal*" efficiency $(H - h)/H$ for an actual engine is still greater than its "*practical*"

efficiency, for all the heat taken is not spent in *useful* work: there is waste work done against friction, etc., and to estimate the practical efficiency this must be deducted. When $(S - T)/S$ is worked out for an actual engine it is called its "*theoretical*" efficiency.

(6) *Influence of Pressure on Temperature of Fusion and Vaporisation.*—A convenient formula for calculations is:—

$$dT = \frac{v \times T \times dP}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

where (in the case say of fusion), L = latent heat of fusion *in dynamical units*, v = difference in the volume *of unit mass* in the solid and liquid states, T = absolute temperature of melting point, dT = change in temperature of fusion due to change of pressure dP. The letters will have their corresponding meanings in vaporisation. Theoretically dP should be *infinitesimally* small, but in fusion the formula holds even for fairly large changes: in vaporisation dP *must* be small, but not necessarily infinitesimally small.

(7) **Entropy Changes.**—(*a*) *General statement.*—If H_1, H_2, H_3, etc., be quantities of heat (in mechanical units) received by a body at absolute temperatures T_1, T_2, T_3, etc., the total change in entropy of the body is

$$\left(\frac{H_1}{T_1} + \frac{H_2}{T_2} + \frac{H_3}{T_3} + \text{etc.}\right).$$

In practical engineering H is more often expressed in thermal units and the entropy is said to be in *ranks*: thus if a body receives 1000 thermal units at a temperature of 500 its entropy increase is 2 ranks.

(*b*) *Change in entropy of a gas due to an isothermal change.*—If say unit mass of a gas at pressure P_1 and absolute temperature T_1 *expands* isothermally from volume V_1 to volume V_2 the *increase* in entropy is

$$\phi_2 - \phi_1 = \frac{P_1 V_1}{T_1} \log_e \frac{V_2}{V_1} = (\gamma - 1)\, J.s_v.\, \log_e \frac{V_2}{V_1} \ldots (10)$$

where γ = ratio of the specific heat at constant pressure to that at constant volume (s_v) and J = mechanical equivalent of heat. In isothermal *contraction* this gives the *decrease* in entropy. Along an *adiabatic* the entropy is constant.

(*c*) *Change in entropy of a substance due to a change in temperature.*—The most convenient formula (for unit mass) is:—

$$\phi_2 - \phi_1 = J.s \log_e \frac{T_2}{T_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where s = the specific heat and T_2, T_1 = final and initial absolute temperatures.

(*d*) *Change in entropy of a gas due to a change from volume V_1 and absolute temperature T_1 to volume V_2 and absolute temperature T_2.*—In the case of a gas the heat goes partly in raising the temperature and partly in doing external work, the latter being appreciable owing to the marked expansion. The result follows from the preceding and the student will readily see that (for unit mass):—

$$\phi_2 - \phi_1 = J.s_v. \log_e \frac{T_2}{T_1} + J\ (s_p - s_v) \log \frac{V_2}{V_1}$$

$$= J.s_v. \log \frac{T_2}{T_1} + (\gamma - 1)\, J.s_v. \log \frac{V_2}{V_1} \quad \ldots\ldots\ldots (12)$$

(*e*) *Change in entropy of a substance due to a change in state.*—In this case, let m = mass of a liquid, vaporised at absolute temperature T, and L = latent heat of the vapour, then the *increase* in entropy is:—

$$\frac{JmL}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

In condensation this gives the *decrease* in entropy. The same formula holds for melting and solidification.

(8) **Second Law of Thermodynamics.**—For reference, four forms of this law are summarised below. As given in the first three forms the first law of thermodynamics is not involved: the fourth form involves the idea of the first law, however.

(*a*) *Carnot's form:* It is impossible for any mechanism to transfer heat from a body at a lower temperature to one at a higher temperature except by the expenditure of external work on it.

(*b*) *Clausius's form:* It is impossible for a self-acting machine unaided by any external agency to convey heat from one body to another at a higher temperature.

(*c*) *Kelvin's form :* It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of surrounding objects.

(*d*) *Maxwell's form :* It is impossible by the unaided action of natural processes to transform any part of the heat of a body into mechanical work except by allowing heat to pass from that body into one at a lower temperature.

It may hurt the feelings of the student to say that, as far as actual heat engines are concerned, the second law means that the boiler must be hotter than the condenser.

(9) **The Four Thermodynamic Relations.**—For reference, these also are summarised below :—

First Thermodynamic Relation : Isothermal entropy-increase per unit volume-increase = Isometric pressure-increase per unit temperature-increase, *i.e.*

$$\left(\frac{d\phi}{dv}\right)_{\text{T constant}} = \left(\frac{dp}{d\text{T}}\right)_{v\text{ constant.}}$$

Second Thermodynamic Relation : Isothermal entropy-*decrease* per unit pressure-increase = Isopiestic volume-increase per unit temperature-increase, *i.e.*

$$\left(-\frac{d\phi}{dp}\right)_{\text{T constant}} = \left(\frac{dv}{d\text{T}}\right)_{p\text{ constant.}}$$

Third Thermodynamic Relation : Isentropic temperature-*decrease* per unit volume-increase = Isometric pressure-increase per unit entropy-increase, *i.e.*

$$\left(-\frac{d\text{T}}{dv}\right)_{\phi\text{ constant}} = \left(\frac{dp}{d\phi}\right)_{v\text{ constant.}}$$

Fourth Thermodynamic Relation : Isentropic temperature-increase per unit pressure-increase = Isopiestic volume-increase per unit entropy-increase.

$$\left(\frac{d\text{T}}{dp}\right)_{\phi\text{ constant}} = \left(\frac{dv}{d\phi}\right)_{p\text{ constant.}}$$

√**Worked Examples.**—(1) *Four cubic feet of air at* 0° *C. and* 76 *cm. pressure are compressed isothermally to* 1 *cubic foot. Find the final pressure and the work done during the change (in gram centimetres).*

(*a*) $$\text{P}_1\text{V}_1 = \text{P}_2\text{V}_2.$$

$$\therefore\ \text{P}_2 = \frac{\text{P}_1\text{V}_1}{\text{V}_2} = \frac{76 \times 4}{1} = 304 \text{ cm.}$$

(*b*) $$\text{W} = \text{P}_1\text{V}_1 \times 2{\cdot}3026 \times \log\frac{\text{V}_1}{\text{V}_2}.$$

To obtain the work in gram-centimetres, P_1 must be in grams per sq. cm. and V_1 in cubic cm.

$$\therefore\ W = (76 \times 13{\cdot}6) \times (4 \times 28320) \times 2{\cdot}3026 \times \log \tfrac{4}{1}.$$
$$= 76 \times 13{\cdot}6 \times 4 \times 28320 \times 2{\cdot}3026 \times {\cdot}6021$$
$$= 1{\cdot}62 \times 10^8 \text{ gram-centimetres (approx.)}.$$

(2) *Two cubic feet of air at 0° C. and 76 cm. pressure are compressed adiabatically to half the volume. Find the final pressure and temperature, and the work done.* ($\gamma = 1{\cdot}41$.)

(*a*)
$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$\therefore\ 76 \times 2^{\gamma} = P_2 \times 1^{\gamma}$$
$$\log 76 + \gamma \log 2 = \log P_2$$
$$\log P_2 = 1{\cdot}8808 + (1{\cdot}41 \times {\cdot}3010)$$
$$= 1{\cdot}8808 + {\cdot}4244 = 2{\cdot}3052.$$
$$\therefore\ P_2 = 202 \text{ cm.}$$

(*b*)
$$T_1V_1^{\gamma - 1} = T_2V_2^{\gamma - 1}$$
$$\therefore\ 273 \times 2^{\gamma - 1} = T_2 1^{\gamma - 1}$$
$$\log 273 + (\gamma - 1) \log 2 = \log T_2$$
$$\log T_2 = 2{\cdot}4362 + ({\cdot}41 \times {\cdot}3010)$$
$$= 2{\cdot}4362 + {\cdot}1234$$
$$= 2{\cdot}5596.$$
$$\therefore\ T_2 = 362{\cdot}7^\circ \text{ abs.} = 89{\cdot}7^\circ \text{ C.}$$

(*c*)
$$W = \frac{P_2V_2 - P_1V_1}{\gamma - 1}.$$

As in Example (1), to obtain the work in gram-centimetres we have:—

$$W = \frac{(202 \times 13{\cdot}6) \times (1 \times 28320) - (76 \times 13{\cdot}6) \times (2 \times 28320)}{{\cdot}41}$$
$$= \frac{13{\cdot}6 \times 28320 \times 50}{{\cdot}41} = 4{\cdot}7 \times 10^7 \text{ gram-cm.}$$

As further practice, the reader should obtain the solution for P_2 in pounds per sq. inch and W in foot-pounds. In (*a*) $P_1 = 14{\cdot}7$ lb. per sq. inch, and the solution becomes :—

$$14{\cdot}7 \times (2)^{1{\cdot}41} = P_2 \times (1)^{1{\cdot}41}$$

which gives $P_2 = 39{\cdot}065$ pounds per sq. inch. In (*c*), if W is to be in foot-pounds, P_1 and P_2 must be in pounds per sq. foot, and V_1 and V_2 in cubic ft., and the solution becomes :—

$$W = \frac{(39{\cdot}065 \times 144) \times 1 - (14{\cdot}7 \times 144) \times 2}{{\cdot}41},$$

which gives W = 3394 foot-pounds.

(3) *Steam just saturated at* 120° *C. expands adiabatically, and the temperature falls to* 100° *C. How much of the total mass of steam is condensed?*

Using formula (7) we have :—

Case 1.—$x_1 = 1$, $L_1 = 606{\cdot}5 - ({\cdot}695 \times 120) = 523$

$T_1 = 273 + 120 = 393$, $s = 1$.

$$\therefore \frac{x_1 L_1}{T_1} + s \log_e T_1 = \tfrac{523}{393} + \log_e 393 \quad \ldots\ldots\ldots (a)$$

Case 2.—x_2 is required. $L_2 = 606{\cdot}5 - ({\cdot}695 \times 100) = 537$.

$T_2 = 273 + 100 = 373$, $s = 1$.

$$\therefore \frac{x_2 L_2}{T_2} + s \log_e T_2 = \frac{x_2 \,.\, 537}{373} + \log_e 373 \quad \ldots\ldots (b)$$

Now, $(a) = (b)$,

$$\therefore \tfrac{537}{373} x_2 + \log_e 373 = \tfrac{523}{393} + \log_e 393.$$

$$\therefore 1{\cdot}4 x_2 = 1{\cdot}33 + \log_e \tfrac{393}{373} = 1{\cdot}33 + {\cdot}0522,$$

i.e. $1{\cdot}4 x_2 = 1{\cdot}3822$ or $x_2 = {\cdot}98$ approx.

Thus $(1 - {\cdot}98)$, *i.e.* ·02 of the total mass is condensed.

(4) *Find the theoretical efficiency of an engine in which the steam pressure in the boiler is* 180 *lb. per square inch, the steam being ejected from the cylinder at a temperature of* 100° *C.*

The steam temperature corresponding to a pressure of 180 lb. per sq. inch is (from tables) 188° C.

$\therefore$ Temperature of boiler $= 188°$ C. $= 461$ abs.

and temperature of "condenser" $= 100°$ C. $= 373$ abs.

$$\therefore \text{"Theoretical" efficiency} = \frac{S - T}{S} = \frac{461 - 373}{461} = \frac{88}{461}$$

$$= 19{\cdot}1 \text{ per cent. (approx.).}$$

The "thermal" efficiency would be about 10 per cent.

(5) *If a steam engine works between 320° F. and 130° F., and the ratio of its efficiency to that of a Carnot's perfect engine is ·55, how many B.Th.U. would it require per horse-power per minute?*

Efficiency of the Carnot's engine

$$= \frac{S - T}{S} = \frac{(461 + 320) - (461 + 100)}{461 + 320}$$

$$= \frac{320 - 100}{461 + 320} = 28{\cdot}2 \text{ per cent.}$$

Efficiency of given engine $=$ ·55 of $28{\cdot}2 = 15{\cdot}5$ per cent.

Now, 1 horse-power $= 33000$ ft.-pounds per minute

$= \frac{33000}{778} = 42{\cdot}4$ B.Th.U. per minute.

$\therefore$ Engine requires $42{\cdot}4 \times \frac{100}{15{\cdot}5}$, *i.e.* 274 B.Th.U.

(6) *The pressure of water vapour at 100° C. is 76 cm.; at 99·9° C. it is 75·7286 cm., and the density of steam at 100° C. and 76 cm. is 1/1668·5. By considering the work done when a gram of water is turned into steam at 100° C. and condensed at 99·9° C., show that the latent heat of vaporisation of water at 100° is about 536.*

$\frac{1}{1668{\cdot}5}$ grm. of steam at 100° C. and 76 cm. pressure $= 1$ c.cm.

$\therefore$ 1 grm. of steam at 100° C. $= 1668{\cdot}5$ c.cm.

and 1 grm. of water at 100° C. $= 1$ c.cm. (approx.).

$\therefore$ Increase in vol. when 1 grm. of water is vaporised at 100° C. $= 1667{\cdot}5$ c.cm.

Now, the difference in the pressures at 100° C. and 99·9° C. $=(76-75\cdot7286)=\cdot2714$ cm. mercury $=(\cdot2714\times13\cdot6\times981)$ $=36260$ dynes. Hence, if a gram of water be turned into steam at 100° C. and condensed at 99·9° C., the work done $=(36260\times1667\cdot5)=6046400$ ergs.

But:—

$$\frac{\text{Work done} \left(= dp \times dv\right)}{\text{Heat (in mechanical units) taken in at } 100^\circ \text{ C.}} = \frac{S-T}{S}.$$

∴ Heat taken in at 100° C. (in mechanical units)

$$= \frac{S}{S-T} \times \text{work done}$$

$$= \frac{373}{\cdot1} \times 6046400 = 22553 \times 10^6 \text{ ergs.}$$

But the heat taken in by the 1 gram of water at 100° C. = the latent heat.

$$\therefore \text{ Latent heat} = \frac{22553 \times 10^6}{4\cdot2 \times 10^7} = 535\cdot74 \text{ calories.}$$

(7) *The change of volume of* 1 *gram of sulphur on melting is* ·027 *c.c. Its melting point is* 115° *C., and its latent heat of fusion is* 9·4. *Find the change in melting point due to a pressure of* 100 *atmospheres.*

Applying (9) we have in this case:—

$$L = 9\cdot4 \times 4\cdot2 \times 10^7 \text{ ergs}; \; v = \cdot027 \text{ c.cm.};$$

$$T = 388^\circ \text{ abs.}; \; dP = 100 \times 1033 \times 981 \text{ dynes per sq. cm.}$$

$$\therefore dT = \frac{\cdot027 \times 388 \times 100 \times 1033 \times 981}{9\cdot4 \times 4\cdot2 \times 10^7}$$

$$= 2\cdot7^\circ \text{ C.}$$

(8) *Calculate the latent heat of steam at* 100° *C. given that an increase of pressure of* ·00364 *megadyne per sq. cm. raises the boiling point from* 100° *C. to* 100·1° *C., that the volume of* 1 *gram of steam at* 100° *C. is* 1645·55 *c.c., and that the volume of* 1 *gram of water at* 100° *C. is* 104 *c.c.*

Here $dT = \cdot 1^\circ$ C.; $v = 1644{\cdot}51$ c.c.;

$T = 373^\circ$ abs.; $dP = \cdot 00364 \times 10^6$ dynes per sq. cm.

$$\therefore \cdot 1 = \frac{1644{\cdot}51 \times 373 \times \cdot 00364 \times 10^6}{L \times 4{\cdot}2 \times 10^7}.$$

$$\therefore 4{\cdot}2L = 1644{\cdot}51 \times 373 \times \cdot 00364$$

$$L = \frac{1644{\cdot}51 \times 373 \times \cdot 00364}{4{\cdot}2}$$

$$= 531{\cdot}6.$$

(9) *Find the change in entropy of a cubic foot of air at N.T.P. in expanding isothermally to double the volume. (Take $\gamma = 1{\cdot}41$ and $J = 1390$ foot-pounds).*

From (10) we have:—

$$\phi_2 - \phi_1 = \left\{ (\gamma - 1)\, Js_v \log_e \frac{V_2}{V_1} \right\} \times m,$$

where m = mass of 1 c.ft. of air at N.T.P. = $\cdot 0807$ pound. Further, $s_v = \cdot 1688$: hence:—

$$\phi_2 - \phi_1 = (\cdot 41 \times 1390 \times \cdot 1688 \times \log_{10} 2 \times 2{\cdot}3026) \times \cdot 0807$$
$$= \cdot 41 \times 1390 \times \cdot 1688 \times \cdot 3010 \times 2{\cdot}3026 \times \cdot 0807$$
$$= 5{\cdot}38 \text{ (using foot-pound-degree-C. units).}$$

(10) *A certain mass of gas is caused to expand isothermally to N times its volume. The same mass of the gas under the same initial conditions of temperature, pressure, and volume is then caused to expand at constant pressure (isopiestically) to N times its volume. Compare the changes in entropy in the two cases. ($\gamma = 1{\cdot}4$.)*

Case 1.—*Isothermal.*

If m be the mass of the gas:—

$$\text{Change in entropy} = \left\{(\gamma - 1)\, Js_v \log_e \frac{V_2}{V_1}\right\} \times m$$
$$= (\gamma - 1) \times J \times s_v \times \log_e N \times m.$$

Case 2.—*Isopiestic.*

Use the formula:—

$$\text{Change in entropy} = \left\{Js \log_e \frac{T_2}{T_1}\right\} \times m.$$

The specific heat in this case is that at constant pressure, viz. γs_v, where s_v, as above, is the specific heat at constant volume. Further, in this case (change in volume of gas at constant pressure) $T_2/T_1 = V_2/V_1 = N$.

$$\therefore \text{ Change in entropy} = J \times \gamma s_v \times \log_e N \times m.$$

Hence:—

$$\frac{\text{Entropy change in Case 1 (isothermal)}}{\text{Entropy change in Case 2 (isopiestic)}} = \frac{\gamma - 1}{\gamma} = \frac{\cdot 4}{1 \cdot 4} = \cdot 28.$$

Exercises XI.

(1) Find the work done in compressing isothermally a cubic foot of air at normal temperature and pressure until it occupies half a cubic foot. Find also the heat which must be communicated to a cubic foot of air at normal temperature and pressure if it expands isothermally to a volume of two cubic feet. (Normal pressure = 14·7 lb. to the square inch; normal temperature = 0° C.).

(2) A cubic metre of air is at 27° C. and 1·0333 kilogrammes per sq. cm. pressure. Find by how much the volume must be increased in order that the pressure may be reduced to one half, the change taking place adiabatically. Calculate also the final temperature of the gas and the work done during the change.

(3) What is the maximum possible efficiency of a reversible engine whose boiler is at 140° C. and condenser at 30° C.?

(4) What is the maximum possible efficiency of a heat engine which has as a source of heat a mass of water at 100° C., and as a refrigerator a mass of water at 20° C.? If this engine takes 500 units of heat from the source, how many units does it return to the refrigerator?

(5) Find the efficiency of an engine using 1 lb. of coal per horse-power hour, the calorific value of the coal being 8,000 calories per gram. Compare this with the efficiency of a Carnot's perfect engine, in which the source is at 1,000° C. and the refrigerator at 0° C.

(6) During a Carnot's cycle of operations the working substance gave out 1,000 pound-degree-C. units of heat and did 160,000 foot-pounds of work: find the amount of heat taken in at the source.

(7) Would more work be obtained from 10 grams of a working substance cooled from 100° C. to 0° C., or from 1,000 grams cooled from 1° C. to 0° C.? Compare the work in the two cases, assuming each to be a perfect reversible engine.

(8) The melting-point of phosphorus is 44° C. and its latent heat of fusion is 5·2. The density of the solid form is 1·83 grm. per c.cm., and 1 c.cm. of the solid at 0° C. expands to 1·017 c.cm. at the melting point, and becomes 1·052 c.cm. on melting. Calculate the change in the melting point due to a change of pressure of 10 atmospheres.

(9) Find the mass of water deposited when a pound of steam exactly saturated at 200° C. is allowed to expand adiabatically to 0° C.

(10) Being given that the latent heat of water = 80, and that the density of ice (water = 1) = 0·918, find the increase of pressure required to cause a lowering of 1° C. in the melting point of ice.

(11) Determine by how much the ordinary boiling point of water is raised if the pressure is increased by ·1 atmosphere.

(12) Find the change in entropy of a cubic foot of air, originally at a pressure of 14·7 lb. to the square inch and a temperature of 0° C., which is allowed to expand isothermally to occupy three times its volume.

(13) A quantity of water is first heated from 0° C. to 100° C., and is then vaporised under atmospheric pressure. Compare the increases of entropy in the two processes.

(14) A certain volume of air at 0° C, and atmospheric pressure is suddenly compressed to $\frac{1}{10}$ of its original volume. Find the momentary pressure and temperature at the end of this process and the work done. (Take $\gamma = 1·408$ and $\log_{10} 2559 = 3·408$.)

(15) A mass of saturated steam has a dryness fraction of ·9 and is at a pressure of 160 inches of mercury. It expands adiabatically to atmospheric pressure. Find the new dryness fraction.

(16) Determine the change in the entropy of 1,000 grams of water on freezing at 0° C.

CHAPTER XII.

MISCELLANEOUS PROBLEMS IN HEAT ENGINES.

Note.—In this chapter formulae are introduced as required. They are, however, not given merely to enable the problem to be solved. *All are fundamental and important, and must be remembered.*

Worked Examples.—(1) *What weight of air must be supplied for the complete combustion of* 1 *lb. of carbon?*

The "composition" of the atmosphere is usually taken as :—

4 parts of nitrogen (N) to 1 part of oxygen (O) by volume,
7 ,, ,, ,, ,, 2 parts ,, ,, ,, weight.

More exactly :—

79 parts of nitrogen (N) to 21 parts of oxygen (O) by volume,
77 ,, ,, ,, ,, 23 ,, ,, ,, ,, weight.

In the problem :— $C + O_2 = CO_2$,
and, taking the atomic weights of carbon and oxygen as 12 and 16 respectively, we have :—

12 lb. of carbon require $(16 \times 2) = 32$ lb. of oxygen.

$\therefore$ 1 lb. ,, ,, requires $\frac{32}{12}$ lb. of oxygen.

But 23 lb. of oxygen mean $77 + 23 = 100$ lb. of air.

$\therefore$ 1 lb. of oxygen means $\frac{100}{23}$ lb. of air.

$\therefore$ $\frac{32}{12}$ lb. of oxygen mean $\frac{100}{23} \times \frac{32}{12} = 11{\cdot}6$ lb. of air.

Hence 1 lb. of carbon requires 11·6 lb. of air: in practice something of the order of 20 lb. would be required.

(2) *Find the Calorific Value of* 1 *lb. of ethylene or olefiant gas.* (*Calorific Values of hydrogen and carbon per lb.* = 61,260 *and* 14,540 *B.Th.U. respectively*). *Olefiant Gas* = C_2H_4.

In the case of olefiant gas there are $(2 \times 12) = 24$ lb. of carbon and $(4 \times 1) = 4$ lb. of hydrogen in 28 lb. of the gas: hence in 1 lb. of the gas there are $\frac{6}{7}$ lb. of carbon and $\frac{1}{7}$ lb. of hydrogen.

Heating value of $\frac{6}{7}$ lb. of carbon $= \frac{6}{7} \times 14{,}540 = 12{,}463$ B.Th.U.

Heating value of $\frac{1}{7}$ lb. of hydrogen $= \frac{1}{7} \times 61{,}260 = 8751$ B.Th.U.

$\therefore$ Calorific value of 1 lb. of the gas $C_2H_4 = 21{,}214$ B.Th.U.

(3) *A certain Welsh coal has the following percentage composition:—Carbon, 87·49; Hydrogen, 3·66; Oxygen, 2·69; Nitrogen, 1·17; Sulphur, ·79; the remainder being ash and moisture. Find the calorific value of the coal per lb.*

In calculating the Calorific Value of any fuel, the various chemical constituents of the fuel are considered separately. The following figures show the number of B.Th.U. obtained from the combustion of 1 lb. of the elements met with in practice:—

Carbon (C)—burnt to CO_2 = 14,540 : Carbon—burnt to CO = 4,450 : Hydrogen (H) = 61,260 : Sulphur (S) = 4,000.

From these figures the following formula is obtained:—

Calorific value per pound of fuel (B.Th.U.)

$$= \mathbf{14{,}540\,C + 61{,}260\,(H - O/8) + 4{,}000\,S},$$

the quantity O/8 being subtracted from H because O/8 of the H will be in the form of water, and therefore (H − O/8) is the H available for heating.

In the case of the given coal, we have *per lb.*, C = 87·49/100 = ·8749 lb., H = ·0366 lb., O = ·0269 lb., S = ·0079 lb.

$\therefore$ Calorific Value

$$= (14540 \times \cdot 8749) + 61260\left(\cdot 0366 - \frac{\cdot 0269}{8}\right) + (4000 \times \cdot 0079)$$

$$= 14{,}790 \text{ B.Th.U. per lb.}$$

(4) *The percentage analysis by volume of a producer gas is $H = 16$, $CO = 20$, $CO_2 = 6$, $N = 58$. Determine the Calorific Value per c.ft. at standard temperature and pressure. Calorific Value of 1 lb. of carbon burning to $CO_2 = 14{,}500$; to CO, 4,400; of $H = 62{,}000$ B.Th.U. The volume of 2 lb. of H at standard temperature and pressure = 357 c.ft.)*

Weight of 1 cu.ft. of H $= 2/357$ lb. $= \cdot 00541$ lb.

100 cu.ft. of gas contain 16 cu.ft., *i.e.* $16 \times \cdot 00541$ lb. of H.

i.e. 1 cu.ft. of gas contains $\dfrac{16 \times \cdot 00541}{100}$ lb. of H.

$$\left.\begin{array}{l}\text{Calorific value of H}\\ \text{per cu.ft. of gas}\end{array}\right\} = \frac{16 \times \cdot 00541 \times 62000}{100} = 53 \cdot 6 \text{ B.Th.U.}$$

Again, since the relative densities of compound gases are proportional to half their molecular weights, and the molecular weight of carbon monoxide (CO) is 28, we have :—

$$\text{Weight of CO per cu.ft.} = \cdot 00541 \times 14 = \cdot 0757 \text{ lb.}$$

Further, since 12 lb. of carbon make 28 lb. of CO, 1 lb. of CO requires $\frac{12}{28}$ lb. of carbon.

$$\therefore \text{ Calorific value of CO per lb.} = \tfrac{12}{28}\,(14500 - 4400)$$
$$= 4{,}330 \text{ B.Th.U. nearly.}$$

$$\therefore \left.\begin{array}{l}\text{Calorific value of CO}\\ \text{per cu.ft. of gas}\end{array}\right\} = \frac{\cdot 0757 \times 4330 \times 20}{100} = 65 \cdot 7 \text{ B.Th.U.}$$

$$\therefore \left.\begin{array}{c}\text{Total calorific value of gas}\\ \text{per cu.ft.}\end{array}\right\} = 53 \cdot 6 + 65 \cdot 7 = 119 \cdot 3 \text{ B.Th.U.}$$

(5) *A boiler shell is* 8 *feet in diameter, and the metal is* $\frac{1}{2}$ *inch thick. If the steam pressure is* 120 *pounds per square inch, calculate the stress on circumferential and longitudinal sections.*

The stress in lb. per square inch on a circumferential section of a boiler shell is given by the formula $\dfrac{pD}{4t}$, where D = internal diameter of shell in inches; p = fluid pressure in lb. per square inch; and t = thickness of plate in inches. The stress on a longitudinal section of the shell $= \dfrac{pD}{2t}$ lb. per sq. inch, *i.e.* it is twice the stress on a circumferential section. The force tending to burst a boiler along its length is twice that tending to blow the ends off.

In the example given, stress on circumferential section $= \dfrac{120 \times 96}{4 \times \frac{1}{2}} = 5{,}760$ lb. per sq. in. Stress on longitudinal section $= 2\,(5760) = 11{,}520$ lb. per sq. in.

(6) *Compressed air at 120 lb. per sq. inch absolute is drawn into a cylinder, and is then expanded to 6 times its original volume. Determine the mean absolute pressure (a) if the temperature is constant, (b) if the cylinder is non-conducting and $\gamma = 1{\cdot}408$. If the initial temperature is 70° F., find the final temperature in the latter case.*

This can be worked out by the formulae of Chapter XI., thus :—

(*a*) *Isothermal Expansion* (see diagram, p. 100).

$$\text{Mean pressure} = p_m = \frac{\text{Area of diagram}}{\text{AB}}$$

$$= \frac{P_1V_1 + P_1V_1 \log_e \frac{V_2}{V_1}}{V_2},$$

i.e. $$p_m = \frac{P_1V_1}{V_2}\left(1 + \log_e \frac{V_2}{V_1}\right) = \frac{120 \times 1}{6}(1 + \log_e 6)$$

$$= 20\,(1 + 1{\cdot}79) = 55{\cdot}8 \text{ lb. per sq. inch.}$$

(*b*) *Adiabatic Expansion* (see diagram).

$$\text{Mean pressure} = p_m = \frac{\text{Area of diagram}}{\text{AB}} = \frac{P_1V_1 + \dfrac{P_1V_1 - P_2V_2}{\gamma - 1}}{V_2}$$

$$= \frac{P_1V_1}{V_2}\left(1 + \frac{1 - \dfrac{P_2V_2}{P_1V_1}}{\gamma - 1}\right)$$

$$\text{Now :—}\ \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \quad \therefore\ \frac{P_2V_2}{P_1V_1} = \frac{V_1{}^{\gamma}V_2}{V_2{}^{\gamma}V_1} = \left(\frac{V_2}{V_1}\right)^{1-\gamma}.$$

$$\text{Hence :—}\ p_m = \frac{P_1V_1}{V_2}\left(1 + \frac{1 - \left(\frac{V_2}{V_1}\right)^{1-\gamma}}{\gamma - 1}\right)$$

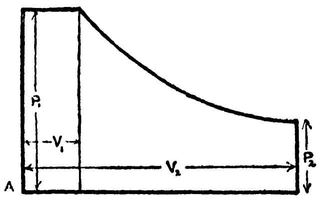
$$= \frac{120 \times 1}{6}\left(1 + \frac{1 - 6^{-\cdot 408}}{\cdot 408}\right) = 20\left(1 + \frac{1 - \cdot 48}{\cdot 408}\right)$$

$$= 45{\cdot}4 \text{ lb. per sq. inch.}$$

(c) *Final Temperature in (b) :—*

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \quad \therefore \quad \frac{T_2}{70 + 461} = \frac{1}{6^{\cdot 408}} = \frac{1}{2{\cdot}08}$$

$$\therefore T_2 = \frac{531}{2{\cdot}08} = 254{\cdot}6^\circ \text{ F. abs.} = -\ 205{\cdot}4^\circ \text{ F.}$$



(7) *Steam enters a cylinder at 180 lb. pressure per sq. inch (absolute) is cut off at one-third of the stroke and expands according to the law pv = constant. The back pressure is 17 lb. (absolute) per sq. inch, the area of the cylinder is 112 sq. inches, and the stroke 24 inches. Find the mean effective pressure, the average pressure absolute during the forward stroke, and the work done in one stroke.*

The mean effective pressure is, in practice, obtained from the indicator diagram. A formula also frequently employed is as follows :—

$$p_m = p_1 \left(\frac{1 + \log_e r}{r}\right) - p_b$$

where p_m = mean effective pressure of steam, p_1 = initial absolute pressure or pressure on admission to cylinder, r = expansion ratio, *i.e.* ratio of volume at end of stroke to volume at point of cut off and p_b = absolute back pressure. The formula assumes the expansion to be pv = constant. In practice, p_m is really less than the value obtained above, and for greater accuracy the expression on the right must be multiplied by a factor less than unity called the "diagram factor."

(a) *Mean effective pressure.* Applying the above formula we have :—

$$p_m = 180\left(\frac{1 + \log_e 3}{3}\right) - 17 = 60\,(1 + 1{\cdot}0986) - 17$$

$$= 109 \text{ lb. per sq. inch approx.}$$

(*b*) *Mean absolute pressure.* Assuming p_b constant the mean absolute pressure is $109 + 17 = 126$ lb. per sq. inch.

(*c*) *Work done per stroke.* Here $p_m = 109$ lb. per sq. inch, Area of piston = 112 sq. inches, Stroke = 2 feet. $\therefore$ Total pressure = (109×112) lb.

$\therefore$ Work done = $(109 \times 112) \times 2 = 24{,}416$ foot pounds.

(8) *A steam engine has a cylinder* 20 *inches in diameter, the crank measures* 18 *inches from the centre of the crank shaft to the centre of the crank pin, the engine runs at* 85 *revolutions per minute, and the mean effective pressure of the steam on the piston is* 28 *lb. per sq. inch. Find the I.H.P. If the B.H.P. be* 107·5, *what is the mechanical efficiency?*

The "*indicated horse-power*" (I.H.P.) of a steam engine is given by the formula :—

$$\text{I.H.P.} = \frac{2p_m\text{LAN}}{33000} \qquad [2\ \textbf{PLAN}]$$

where L = length of stroke in feet, A = area of piston in sq. inches, and N = revolutions per minute ($\therefore$ 2 N = number of strokes per minute).

In the case of gas and oil engines we have :—

$$\text{I.H.P.} = \frac{\text{PALE}}{33000} \qquad [\textbf{PALE}]$$

where $P = (p_2 - p_1)$ in which p_2 = mean pressure during the working stroke, p_1 = mean pressure during the compression stroke, and E = number of explosions per minute.

The horse-power as measured from the *useful* work done (*e.g.* driving machinery) is called the "*brake horse-power*" (B.H.P.): it is less than the I.H.P. because from 5 to 25 per cent. of the latter is used up in overcoming frictional resistance in the engine mechanism.

The "*mechanical efficiency*" is the ratio of the B.H.P. to the I.H.P.

In the problem we have :—$p_m = 28$ lb. per sq. inch, $A = \cdot 7854d^2 = (\cdot 7854 \times 20 \times 20)$ sq. inches, N = 85, L = stroke = twice length of crank = 36 inches = 3 ft.

$$\therefore \text{I.H.P.} = \frac{2p_m\text{LAN}}{33000} = \frac{2 \times 28 \times 3 \times \cdot 7854 \times 20 \times 20 \times 85}{33000}$$

$$= 136 \text{ nearly.}$$

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{107\cdot 5}{136}$$

$$= \cdot 79 = 79 \text{ per cent.}$$

Before proceeding further the student should again refer to the various remarks and problems on "efficiency" given in Chapters X. and XI. and should study these in conjunction with the various "efficiency" problems which follow in this chapter. It may be briefly indicated, however, that the user of a plant is concerned to a very great extent with the "financial" aspect of the case, and whilst these various efficiencies are of importance to him, he has to consider, in addition, capital outlay, rent of floor space, running costs, etc., etc., and in fact his aim is to produce energy as cheaply as possible: with him, therefore, "cash per brake horse-power hour" is often the deciding factor in the choice of a plant.

(9) *Determine the indicated horse-power, the brake horse-power, the mechanical efficiency, the thermal efficiency, and the overall efficiency of a gas engine used in driving a dynamo, the following particulars being given :—Net mean pressure = 105 lb. per sq. inch, Revolutions per minute = 164, Cylinder diameter = 16 inches, Stroke = 24 inches, Calorific value of the gas used = 150 B.Th.U. per c. foot, Gas used per hour = 5300 c. feet, Electrical load = 55 kilowatts, Efficiency of dynamo = 87 per cent.*

(*a*) *Indicated horse-power* :—From the formula (assuming say Otto cycle of one explosion per four strokes) we have :—

$$\text{I.H.P.} = \frac{105 \times (\cdot 7854 \times 16^2) \times 2 \times \frac{164}{2}}{33000}$$

$$= \frac{105 \times 201 \times 164}{33000} = 104 \cdot 9.$$

(*b*) *Brake horse-power* :—Electrical load = 55 k.w. = 55000 watts = (55000 ÷ 746) H.P. and the dynamo efficiency is 87 per cent.

$$\therefore \text{B.H.P.} = \frac{55000}{746} \times \frac{100}{87} = 84 \cdot 7.$$

(*c*) *Mechanical efficiency* :—From the definition :—

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{84 \cdot 7}{104 \cdot 9}$$

$$= \cdot 807 = 80 \cdot 7 \text{ per cent.}$$

(*d*) *Thermal efficiency* :—The calorific value of the *gas used per hour* is 150 × 5300 = 795000 B.Th.U., and the *heat appearing as work per hour* is (104·9 × 33000 × 60)/778 = 267000 B.Th.U.

$$\therefore \text{ Thermal efficiency} = \frac{267000}{795000} = 33 \cdot 6 \text{ per cent.}$$

(*e*) *Overall efficiency*:—Here this is the ratio of the electrical load (in H.P.) to the I.H.P.

$$\therefore \text{ Overall efficiency} = \frac{55000}{746} \div 104 \cdot 9 = 70 \cdot 3 \text{ per cent.}$$

(10) *A locomotive has two double-acting cylinders each 17 inches in diameter and stroke 24 inches, and the diameter of the driving wheel is 6 feet. The admission pressure (absolute) of the steam is 160 lb. per sq. inch and the exhaust is 20 lb. per sq. inch. The cut-off is ·4, the diagram factor ·8, the mechanical efficiency 82 per cent., and the engine is running at 40 miles per hour. Determine the I.H.P. and the T.E. (tractive effort).*

From (7) we have:—$p_m = 160 \left(\frac{1 + \log_e 2 \cdot 5}{2 \cdot 5} \right) - 20 =$

$\frac{160}{2 \cdot 5} (1 + \cdot 916) - 20 = (64 \times 1 \cdot 916) - 20 = 102 \cdot 64$ approx.

This is the "theoretical" mean effective pressure: the "actual" mean effective pressure is ·8 of $102 \cdot 64 = 82 \cdot 1$ lb. per sq. inch.

Now a speed of 40 miles per hour $= 58 \cdot 67$ feet per second, and as the diameter of the driving wheel is 6 feet and circumference therefore 6π feet, the number of revolutions of the driving wheel is $58 \cdot 67/6\pi$, *i.e.* 3·11 *per second*. ($\therefore$ Strokes per second $= 6 \cdot 22$). Hence:—

I.H.P. *for one cylinder*

$$= \frac{2 \times 82 \cdot 1 \times 2 \times (\cdot 7854 \times 17^2) \times 3 \cdot 11}{550} = 421.$$

$\therefore$ Effective H.P. *for locomotive*

$$= \left(\frac{82}{100} \text{ of } 421 \right) \times 2 = 690 \cdot 4.$$

Further:—

Work done per second $= (550 \times 690 \cdot 4)$ foot pounds.

$$\therefore \text{ Tractive effort} = \frac{\text{Work per second}}{\text{Distance per second}}$$
$$= \frac{550 \times 690{\cdot}4}{58{\cdot}67} = 6{,}472 \text{ lb. approx.}$$

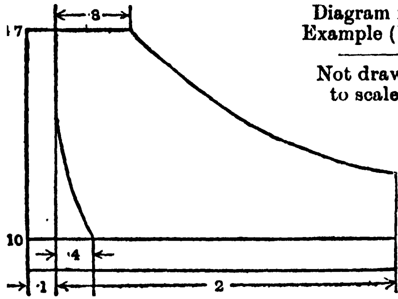


Diagram for Example (11).

Not drawn to scale.

(11) *The stroke of a double acting steam engine is 2 ft. the diameter of the cylinder 15 inches, and the diameter of the piston rod 3 inches. The cut off is ·4 of the stroke, the compression ·2 of the stroke, and the clearance 5 per cent. of the stroke. The boiler pressure is 100 lb. per sq. inch gauge, the exhaust 10 lb. per sq. inch absolute, and the revolutions are 95 per minute. Determine the I.H.P.*

·4 of the stroke = ·4 of 2 ft. = ·8 ft.
·2 „ „ = ·2 of 2 ft. = ·4 ft.
5 per cent. of the stroke = $\frac{5}{100}$ of 2 ft. = ·1 ft.
100 lb. per sq. inch (gauge) = 114·7 lb. (absolute)
} *See diagram.*

∴ Positive area of the card

$$= (114{\cdot}7 \times {\cdot}8) + 114{\cdot}7 \times {\cdot}9 \times \log_e \frac{2{\cdot}1}{{\cdot}9} = 179{\cdot}3 \; (a)$$

and negative area of the card

$$= (10 \times 1{\cdot}6) + 10 \times {\cdot}5 \times \log_e \frac{{\cdot}5}{{\cdot}1} = 24{\cdot}0 \; (b)$$

$$\therefore \text{ Area of card} = (a) - (b) = 179{\cdot}3 - 24{\cdot}0 = 155{\cdot}3.$$

$$\text{Hence:—Mean pressure} = \frac{\text{Area of card}}{\text{Length}} = \frac{155{\cdot}3}{2}$$
$$= 77{\cdot}6 \text{ lb. per sq. inch.}$$

Again:—Area of cylinder $= (\cdot 7854 \times 15^2) = 176\cdot 71$ sq. in.
Cross section of piston rod $= (\cdot 7854 \times 3^2) = 7\cdot 07$ sq. in.

$$\therefore \text{ Mean area} = \frac{176\cdot 71 + (176\cdot 71 - 7\cdot 07)}{2} = 173\cdot 2 \text{ sq. in.}$$

$$\therefore \text{ I.H.P.} = \frac{2p_m LAN}{33000} = \frac{2 \times 77\cdot 6 \times 2 \times 173\cdot 2 \times 95}{33000} = 154\cdot 8.$$

(12) *In a jacket steam engine developing* 645 *H.P. the cylinder feed was* 136·5 *lb. per min. and the jacket feed* 6·2 *lb. Assuming that all jacket water is returned to the feed tank, find the heat supplied to engine cylinders by jackets per H.P. per minute and the thermal efficiency of the engine, if the feed temperature is* 90° *F. and the steam temperature* 373° *F.*

The total heat of steam per lb. at 373° F. = 1195·7 B.Th.U. (from tables), and the feed temperature is 90° F.; hence:—

Heat used per lb. $= 1195\cdot 7 - (90 - 32) = 1137\cdot 7$ B.Th.U.

$\therefore$ Heat supplied by jackets per min. per H.P.

$$= \frac{1137\cdot 7 \times 6\cdot 2}{645} = 10\cdot 9 \text{ B.Th.U.}$$

Again, the total steam used per minute $= 136\cdot 5 + 6\cdot 2 = 142\cdot 7$ lb., which means a heat supply $= (1137\cdot 7 \times 142\cdot 7)$ B.Th.U., and the heat transformed into work per minute $= \frac{645 \times 33000}{778}$ B.Th.U.

$\therefore$ Thermal efficiency

$$= \frac{645 \times 33000}{1137\cdot 7 \times 142\cdot 7 \times 778} = \cdot 167 = 16\cdot 7 \text{ per cent.}$$

(13) *A certain Otto Cycle gas engine uses* 18 *c.ft. of gas per I.H.P. hour. The heating value of the gas is* 620 *B.Th.U. per c.ft. The ratio of compression is* 4·5. *Determine the theoretical efficiency and the actual thermal efficiency (using I.H.P.). The ratio of the specific heats is* 1·36.

The "theoretical" efficiency of an engine on the Otto Cycle depends on the compression ratio (R) and the ratio (γ) of the two specific heats of the gas: it is given by the expression:

$$\text{Theoretical efficiency} = 1 - \frac{1}{R^{\gamma - 1}}.$$

(*a*) *Theoretical efficiency*:—

$$\text{Theor. Effic.} = 1 - \frac{1}{R^{\gamma - 1}} = 1 - \frac{1}{4 \cdot 5^{\cdot 36}} = 1 - \cdot 582$$
$$= \cdot 418 = 41 \cdot 8 \text{ per cent.}$$

(*b*) *Actual thermal efficiency* (*using I.H.P.*):—

B.Th.U. supplied per I.H.P. hour $= 620 \times 18$

Work per I.H.P. hour $= 33000 \times 60$ ft. pounds

B.Th.U. equivalent to this $= (33000 \times 60)/778$

$$\therefore \text{ Actual Thermal Effic.} = \frac{33000 \times 60}{778 \times 620 \times 18}$$
$$= \cdot 228 = 22 \cdot 8 \text{ per cent.}$$

(14) *Steam expands adiabatically in a turbine nozzle. The steam pressure is 200 lb. per sq. inch (gauge) and the counter pressure is 2·4 lb. per sq. inch (absolute). Find the steam velocity. If the steam leaves the turbine with 35 per cent. of its initial velocity determine the number of pounds of steam per H.P. hour.*

In the steam turbine, if M = weight of steam supplied per second (in lbs.), v_1 = initial velocity of steam (in ft. per sec.) and v_2 = leaving velocity of steam (in ft. per sec.), then:—

$$\text{Kinetic energy available per sec.} = \frac{Mv_1^2}{2g} \text{ ft. pounds.}$$

$$\text{Kinetic energy carried away per sec.} = \frac{Mv_2^2}{2g} \text{ ft. pounds.}$$

$$\therefore \text{ Energy for conversion to work per sec.} = \frac{M}{2g}(v_1^2 - v_2^2) \text{ ft. pounds.}$$

And:— $$\text{Available H.P.} = \frac{M}{2g}(v_1^2 - v_2^2)/550.$$

Also:—

$$\text{Efficiency} = \frac{\text{Energy for conversion to work}}{\text{Energy supplied}} = \frac{v_1^2 - v_2^2}{v_1^2} = 1 - \frac{v_2^2}{v_1^2}.$$

The thermal energy supplied *per lb. of steam* can be obtained as follows:—Take from tables the temperatures t_1 and t_2 corresponding to the pressure p_1 of the steam and the counter-pressure p_2; then

$$\text{Thermal energy supplied per lb.} = (t_1 - t_2)\left(1 + \frac{L}{T_1}\right) - T_2 \log_e \frac{T_1}{T_2},$$

T_1 and T_2 being the absolute temperatures corresponding to t_1 and t_2 and L the latent heat at t_1.

For 200 lb. per sq. inch (gauge) :—Temp. $= t_1 = 387{\cdot}6°$ F.
Latent Heat $= \mathrm{L} = 840{\cdot}1$.

For 2·4 lb. per sq. in. (abs.) :—Temp. $= t_2 = 133°$ F.

Now the energy per lb. of steam

$$= (t_1 - t_2)\left(1 + \frac{\mathrm{L}}{\mathrm{T}_1}\right) - \mathrm{T}_2 \log_e \frac{\mathrm{T}_1}{\mathrm{T}_2}$$

$$= 254{\cdot}6\left(1 + \frac{840{\cdot}1}{461 + 387{\cdot}6}\right) - (461 + 133)\log_e \frac{848{\cdot}6}{594}$$

$$= 285 \times 778 \text{ ft. pounds.}$$

Again energy supplied per lb. of steam per sec. $= \dfrac{v_1^2}{2g}$ ft. pounds.

$$\therefore \frac{v_1^2}{2g} = 285 \times 778. \quad \therefore v_1 = \sqrt{2 \times 32 \times 285 \times 778},$$

$$i.e.\ v_1 = 3780 \text{ ft. per sec.}$$

Hence leaving velocity $= v_2 = 3780 \times \frac{35}{100} = 1323$ ft. per sec.

Again work done per lb. of steam per sec. $= \dfrac{v_1^2 - v_2^2}{2g}$ ft. pounds;

Hence if m be the mass of steam per sec. we have :—

$$\text{Work done per sec.} = \left(\frac{3780^2 - 1323^2}{2 \times 32}\right) m \text{ ft. pounds.}$$

$$\therefore \text{ H.P.} = \frac{3780^2 - 1323^2}{2 \times 32 \times 550} m = 356m.$$

Hence $356m$ H.P. means m lb. of steam per sec.

$$\therefore 1 \text{ H.P. means } \frac{m}{356m} \text{ lb. of steam per sec.}$$

$$\therefore 1 \text{ H.P. hour means } \frac{3600}{356},\ i.e.\ 10 \text{ lb. of steam.}$$

(15) *In a test on a steam engine the boiler temperature was 352·7° F. and the hotwell temperature 70·6° F. What is the efficiency on the Rankine-Clausius Cycle?*

Rankine-Clausius Cycle. There are other cycles in addition to the Carnot's cycle referred to in Chapter XI. Thus we have the Stirling, the Ericsson, the Joule Constant Pressure and the Rankine-Clausius cycles. In the latter, steam is taken in at constant pressure, expanded

adiabatically, driven out at constant pressure and condensed and then heated up and evaporated. The Institution of Civil Engineers has chosen this as the ideal Steam Engine Cycle; but it is, of course, not reversible. The efficiency on this cycle is given by the expression :—

$$\text{Efficiency} = \frac{(T_1 - T_0)\left(1 + \frac{L_1}{T_1}\right) - T_0 \log_e \frac{T_1}{T_1}}{L_1 + (T_1 - T_0)} \cdot$$

where T_1 = boiler temperature, T_0 = feed temperature, and L_1 = latent heat at T_1. If the superheat be t the efficiency is given by :—

$$\text{Efficiency} = \frac{(T_1 - T_0)\left(1 + \frac{L_1}{T_1}\right) - T_0 \log_e \frac{T_1}{T_0} + \cdot 48t - \cdot 48 T_0 \log_e \frac{T_1 + t}{T_1}}{L_1 + (T_1 - T_0) + \cdot 48t} .$$

From tables, L at 352·7° F. = 865·6.

Using the above formula :—

R.C. Effic. =

$$\frac{(352 \cdot 7 - 70 \cdot 6)\left(1 + \frac{865 \cdot 6}{461 + 352 \cdot 7}\right) - (461 + 70 \cdot 6) \log_e \frac{461 + 352 \cdot 7}{461 + 70 \cdot 6}}{865 \cdot 6 + (352 \cdot 7 - 70 \cdot 6)}$$

$$= \cdot 312 = 31 \cdot 2 \text{ per cent.}$$

(16) **Heat Balance-Sheet for a Steam Engine.** *The following details of a simple test on a simple engine are given as an elementary illustration of the drawing up of a heat balance-sheet in an engine test.*

The duration of the test was one hour. The I.H.P. was found to be 30·15 and the steam pressure in the valve chest was 75 lb. per sq. inch absolute. The exhaust steam weighed 800 lb. It was discharged at atmospheric pressure, condensed, and cooled to 112° F. 16,000 lb. of condensing water were raised 45° F. in temperature. At the end of the test the slide valve was removed and the exhaust stopped. Steam was then turned on and the amount condensed in the cylinder in one hour was 88·8 lb. This may be taken as a measure of the heat lost during the test by radiation.

Draw up a heat balance-sheet for the engine.

From tables the total heat of 1 lb. of steam at 75 lb. pressure absolute = 1176 B.Th.U.

Steam supplied per minute = 800/60 = $13\frac{1}{3}$ lb.

$\therefore$ Heat supplied per minute $= 13\frac{1}{3} \times 1176 = 15{,}680$ B.Th.U., and heat discharged in exhaust steam per minute

$$= \left(\frac{16000}{60} \times 45\right) + 13\frac{1}{3}(112 - 32) = 13{,}067 \text{ B.Th.U.}$$

Again :—Temperature corresponding to 75 lb. per sq. inch pressure $= 307^\circ$ F. and the latent heat is 901 B.Th.U.

$\therefore$ Heat lost by radiation per minute

$$= \frac{88 \cdot 8}{60} \times 901 = 1{,}334 \text{ B.Th.U.}$$

Finally :—Heat converted into work per minute (using I.H.P.)

$$= \frac{30 \cdot 15 \times 33000}{778} = 1{,}279 \text{ B.Th.U.},$$

Heat Balance Sheet.

	B.Th.U.		B.Th.U.
Heat supplied in steam per minute	15,680	Heat discharged in exhaust per minute ...	13,067
		Heat lost by radiation per minute	1,334
		Heat converted into work per minute ...	1,279
	15,680		15,680

Note.—Many problems in "Heat Engines" belong to "Mechanics" rather than to "Heat." A few are included in Exercises XII.

Exercises XII.

(1) A coal contains 80 per cent. by weight of carbon, 5 of hydrogen, and 10 of oxygen. Calculate its heating value. Calculate approximately the weight of air required for complete combustion of 1 lb. of the coal. What weight of air would be required in practice?

(2) A cylindrical boiler 8 feet in diameter is to withstand a working pressure of 100 lb. per square inch. Calculate to the nearest $\frac{1}{8}$ inch the thickness of the shell, allowing a stress of 10,000 lb. per square inch, and neglecting the effect of the joint.

(3) An engine with a piston 12 inches in diameter and 18 inches stroke was working at 100 revolutions per minute. Indicator diagrams were taken with a $\frac{1}{40}$ spring, and their mean heights were 1·24 and 1·32 inches. What "indicated horse-power" was the engine developing?

(4) Steam at 150 lb. per square inch (absolute) is cut off at $\frac{1}{3}$ stroke, and expands according to the law pv constant. Find the average pressure in the forward stroke. The back pressure is 18 lb. per square inch: what is the effective pressure on the piston? The piston is 15 inches diameter; crank, 1 foot; two strokes in the revolution; 120 revolutions per minute; find the work in one revolution, and the I.H.P. If the mechanical efficiency is 80 per cent., what is the B.H.P.?

(5) The area of a petrol-engine diagram is (using the planimeter which subtracts and adds properly) 4·12 square inches, and its length (parallel to the atmospheric line) is 3·85 inches: what is the average breadth of the figure? If 1 inch represents 70 lb. per square inch, what is the average pressure? The piston is 3·5 inches in diameter, with a stroke of 4 inches. What is the work done in one cycle? If there are 800 cycles per minute, what is the horse-power?

(6) Find the I.H.P., the B.H.P., the Mechanical Efficiency, the Thermal Efficiency, and the Overall Efficiency of a gas engine used in driving a dynamo, the following particulars being given:—Mean pressure = 100 lb. per square inch; revolutions per minute, 160; cylinder diameter, 14 inches; stroke, 24 inches; calorific value of gas = 140 B.Th.U. per cubic foot. Gas used per hour = 5,500 cubic feet. Electrical load = 50 kilowatts. Dynamo efficiency = 85 per cent.

(7) The stroke of a double acting steam engine is 2 feet; cylinder diameter, 15 inches; diameter of piston rod, 2 inches; cut off, ·3 of the stroke; compression, ·1 of the stroke; clearance, 6 per cent. of the stroke. The boiler pressure is 90 lb. per square inch gauge; exhaust, 8 lb. per square inch absolute; and the revolutions 90 per minute. Determine the I.H.P.

(8) In a jacket steam engine, developing 645 H.P., the cylinder feed was 136·5 lb. per minute, and the jacket feed 6·2 lb. Assuming all jacket water returned to the feed tank, find the heat supplied to the engine cylinder by jackets per H.P. per minute, and the thermal efficiency of the engine, if the feed temperature is 90° F. and the steam temperature 373° F.

(9) A triple-expansion engine has cylinders 40″, 60,″ and 96″ in diameter. If the high-pressure valve cuts off at half-stroke, calculate the total ratio of expansion.

The stroke of the above engine is 4·6″, and the mean effective pressures on the three pistons during a certain trial were 59·0, 27·1, and 10·5 lb. per square inch when the shaft was making 80 revolutions per minute. What horse-power was being developed in each cylinder?

(10) An engine gives 10 indicated horse-power and 7·6 brake horse-power for a consumption of 230 cubic feet of coal gas per hour. The calorific power of the gas is 530,000 foot-pounds per cubic foot. What are the mechanical and thermal efficiencies?

(11) Prove that the theoretical efficiency of an engine working on the Otto cycle depends only on the ratio of compression adopted. In an engine using this cycle 16·3 cubic feet of gas are used per I.H.P. hour, the calorific value being 650 B.Th.U. per cubic foot. If the ratio of specific heats at constant pressure and at constant volume is 1·36 and the ratio of compression is 4·5, find the actual and theoretical thermal efficiencies.

(12) What is meant by the term "clearance"? Assuming that the clearance has been reduced to an equivalent length of stroke, which is 4 feet, and taking the case where the cut-off is at $\frac{1}{2}$ stroke, clearance being 3 inches, compare pressures at 3 feet stroke with the above clearance and assuming that there is no clearance. (Take hyperbolic expansion.)

(13) In a steam engine test the following readings were made:—

Speed (revs. per min.)	164
Steam pressure in separator (352·7° F.)	124·6 gauge
Mean dryness fraction	·966
Diameter of brake wheel	3 feet
Diameter of rope	$\frac{1}{2}$ inch
Effective resistance of brake (mean)	147·5 lb.
I.H.P....	7·7
Steam consumption per hour	296 lb.
Hotwell temp.	70·6° F.

Find the mechanical efficiency, thermal efficiency, and efficiency on the Rankine-Clausius Cycle.

(14) A locomotive has two double-acting cylinders supplied by steam, admission pressure 150 lb. per square inch; exhaust 18 lb. per square inch. The cylinder diameters are 17 inches each, and the stroke 26 inches, and the diameter of the driving wheel 6 feet. Find the tractive effort and the indicated horse-power when running at 40 miles per hour, the cut-off being ·4.

Allow a diagram factor of ·9 and a mechanical efficiency 85 per cent.

(15) An air compressor takes 100 cubic feet of free air per minute (temp. = 60° F.) at 200 lb. gauge in two stages with intercooler cooling to 60° F. Find the size of cylinders for an expansion curve $pv^{1 \cdot 2}$ = constant. Piston speed = 300 feet per min. at 200 r.p. minute. Also find H.P. required to drive compressor, taking mechanical efficiency = 90 per cent. Clearance volume = $\frac{1}{10}$ cylinder volume. One pound of air at 60° F. and 14·7 lb. pressure has a volume of 12·8 cubic feet. Find also the temperature at which the air would be delivered.

(16) The travel of a slide is $2\frac{1}{2}''$, the outside lap is $\frac{3}{4}''$, and the lead is $\frac{1}{3}''$. Find the angle of advance, and the crank positions at admission and cut-off.

(17) If a piston with its rod weighs 250 lb.; and if at a certain instant when the resultant force due to steam pressures is 3 tons the piston has an acceleration of 320 feet per second in the same direction, what is the actual force acting on the cross head?

(18) F lb. is the outward radial force on each ball of a governor required to keep it in equilibrium at the distance r feet from the axis when not revolving. The following are for extreme cases:—

r.	F.
0·5	100·1
0·7	144·6

The weight of each ball being 10 lb., what is the centrifugal force of each at n revolutions per minute, the radius being r? What are the speeds for the above values of r when the governor is revolving?

(19) Sketch the construction of a lever safety valve with balance weight, and state under what circumstances such a construction could not be used. If the lever is 16 inches in length, and the centre of the valve seat is 4 inches from the fulcrum, while the diameter of the valve is 4 inches, find the weight to be placed at the end of the lever so that steam may blow off at a pressure of 45 lb. per square inch, the weight of the valve and of the lever being neglected.

(20) Sketch and describe the construction of the air-pump of a condensing engine. What is the use of the air-pump? If the temperature of the injection water supplied to a jet condenser be 62° F., and the water is pumped out of the hot well at a temperature of 106° F., and the steam to be condensed enters the condenser at a temperature of 212° F., what weight of injection water would be required per pound of steam condensed?

APPENDIX.

TABLES.

1.—TABLE OF PRESSURE OF AQUEOUS VAPOUR.

(Regnault).

Temperature. C.	Pressure in mms. of mercury.	Temperature. C.	Pressure in mms. of mercury.	Temperature. C.	Pressure in mms. of mercury.
0°	4·600	50°	91·982	120°	1491·280
5°	6·534	55°	117·478	130°	2030·28
10°	9·165	60°	148·791	140°	2717·63
15°	12·699	65°	186·495	150°	3581·23
20°	17·391	70°	233·093	160°	4651·62
22°	19·659	75°	288·517	170°	5961·66
24°	22·184	80°	354·643	180°	7546·39
25°	23·550	85°	433·041	190°	9442·70
30°	31·584	90°	525·450	200°	11688·96
35°	41·827	95°	633·778	210°	14324·80
40°	54·906	100°	760·000	220°	17390·36
45°	71·391	110°	1075·370	230°	20926·40

2.—TABLE OF PRESSURE AND VOLUME OF SATURATED STEAM.

The following table gives the volume occupied by saturated steam (compared with the water from which it is produced), at the stated temperatures on the Centigrade scale, and pressures given in millimetres of mercury :—

Temperature. C.	Pressure.	Volume.	Temperature. C.	Pressure.	Volume.
70·76°	240·0	4920·2	128·41°	1935·4	649·2
77·18°	316·7	3722·6	130·67°	2070·8	635·3
79·40°	345·9	3438·1	134·86°	2342·6	584·4
83·49°	406·6	3051·0	137·45°	2529·8	515·0
86·83°	466·3	2623·4	139·21°	2655·2	497·2
92·66°	581·2	2149·5	141·80°	2864·6	458·3
117·16°	1361·7	943·1	144·74°	3105·1	433·1
124·16°	1697·7	759·4			

3.—TABLE OF PROPERTIES OF STEAM (FAHRENHEIT UNITS)
(For use in Problems in Applied Heat).

Absolute pressure in lbs. per square inch.	Temperature reading in degrees Fahrenheit	TOTAL HEAT per pound of steam from water at 32° F.	Volume of one pound of steam in cubic feet.	Absolute pressure in lbs. per square inch.	Temperature reading in degrees Fahrenheit.	TOTAL HEAT per pound of steam from water at 32° F.	Volume of one pound of steam in cubic feet.
1	102·0	1113·0	330	46	275·7	1166·0	9·02
2	126·3	1120·5	172	47	277·0	1166·4	8·84
3	141·6	1125·1	118	48	278·3	1166·8	8·67
4	153·1	1128·6	89·6	49	279·6	1167·2	8·50
5	162·4	1131·5	72·5	50	280·9	1167·6	8·34
6	170·2	1133·8	61·1				
7	176·9	1135·9	52·9	55	286·9	1169·5	7·62
8	182·9	1137·7	46·6	60	292·6	1171·2	7·02
9	188·4	1139·4	41·8	65	297·8	1172·8	6·52
10	193·3	1140·9	37·8	70	302·8	1174·3	6·08
				75	307·4	1175·7	5·69
11	197·8	1142·3	34·6	80	311·9	1177·1	5·36
12	202·0	1143·6	31·9	85	316·1	1178·3	5·06
13	205·9	1144·7	29·6	90	320·1	1179·6	4·80
14	209·6	1145·9	27·6	95	323·9	1180·7	4·56
14·7	212·0	1146·6	26·4	100	327·6	1181·9	4·34
15	213·1	1146·9	25·8				
16	216·3	1147·9	24·3	105	331·2	1182·9	4·15
17	219·5	1148·9	23·0	110	334·6	1184·0	3·97
18	222·5	1149·8	21·8	115	337·9	1185·0	3·81
19	225·3	1150·6	20·7	120	341·1	1186·0	3·66
20	228·0	1151·5	19·7	125	344·1	1186·9	3·52
				130	347·1	1187·8	3·39
21	230·6	1152·3	18·8	135	350·0	1188·7	3·27
22	233·1	1153·1	18·0	140	352·8	1189·6	3·16
23	235·5	1153·8	17·3	145	355·6	1190·4	3·06
24	237·8	1154·5	16·6	150	358·2	1191·2	2·96
25	240·0	1155·2	16·0				
26	242·2	1155·9	15·4	155	360·7	1192·0	2·87
27	244·3	1156·5	14·9	160	363·3	1192·8	2·79
28	246·4	1157·1	14·4	165	365·7	1193·5	2·71
29	248·4	1157·7	13·9	170	368·2	1194·2	2·63
30	250·3	1158·3	13·5	175	370·5	1194·9	2·56
				180	372·8	1195·6	2·49
31	252·2	1158·9	13·1	185	375·1	1196·3	2·43
32	254·0	1159·5	12·7	190	377·3	1197·0	2·37
33	255·8	1160·0	12·3	195	379·5	1197·7	2·31
34	257·5	1160·5	12·0	200	381·6	1198·4	2·26
35	259·2	1161·0	11·7				
36	260·9	1161·5	11·4	205	383·7	1199·0	2·21
37	262·5	1162·0	11·1	210	385·8	1199·6	2·16
38	264·1	1162·5	10·8	215	387·8	1200·2	2·11
39	265·7	1163·0	10·5	220	389·8	1200·8	2·06
40	267·2	1163·5	10·3	225	391·8	1201·4	2·02
				230	393·8	1202·0	1·98
41	268·7	1164·0	10·1	235	395·7	1202·5	1·94
42	270·1	1164·4	9·83	240	397·5	1203·0	1·90
43	271·5	1164·8	9·61	245	399·3	1203·5	1·86
44	272·9	1165·2	9·40	250	401·1	1204·0	1·83
45	274·3	1165·6	9·21				

4.—TABLE OF PROPERTIES OF STEAM (CENTIGRADE UNITS).

(For use in Problems in Applied Heat).

Temperature. C.	Pressure lbs. per square inch.	Volume of 1 lb. in cubic feet.	Total Heat.	Latent Heat.
20	0·333	936·9	612·6	592·6
25	0·452	700·8	614·1	589·1
30	0·607	530·7	615·6	585·6
35	0·806	405·9	617·2	582·1
40	1·06	313·6	618·7	578·6
45	1·38	244·6	620·2	575·1
50	1·78	192·5	621·7	571·7
55	2·27	152·8	623·3	568·2
60	2·88	122·3	624·8	564·7
65	3·62	98·7	626·3	561·1
70	4·51	80·23	627·8	557·6
75	5·58	65·64	629·4	554·1
80	6·86	54·06	630·9	550·6
85	8·38	44·81	632·4	547·1
90	10·16	37·36	633·9	543·6
95	12·26	31·34	635·5	540·0
100	14·70	26·43	637·0	536·5
105	17·53	22·40	638·5	533·0
110	20·80	19·08	640·9	529·4
115	24·54	16·32	641·6	525·8
120	28·83	14·04	643·1	522·3
125	33·71	12·12	644·6	518·7
130	39·25	10·51	646·1	515·1
135	45·49	9·147	647·7	511·6
140	52·52	7·995	649·2	508·0
145	60·40	7·009	650·7	504·4
150	69·21	6·168	652·2	500·8
155	79·03	5·446	653·8	497·2
160	89·86	4·827	655·3	493·5
165	101·9	4·290	656·8	489·9
170	115·1	3·823	658·3	486·3
175	129·8	3·419	659·9	482·7
180	145·8	3·065	661·4	479·0
185	163·3	2·756	662·9	475·3
190	182·4	2·482	664·4	471·7
195	203·3	2·242	666·0	468·0
200	225·9	2·031	667·5	464·3
205	250·3	1·843	669·0	460·6

5.—TABLE OF HYPERBOLIC OR NAPIERIAN LOGARITHMS.

No.	Log.	No.	Log.	No.	Log.
1·11	0·1043	2·5	0·9162	8	2·0795
1·143	0·1335	2·66	0·9784	9	2·1971
1·25	0·2231	3	1·0986	10	2·3026
1·33	0·2853	3·33	1·2029	11	2·3979
1·43	0·3576	4	1·3864	12	2·4850
1·6	0·4700	5	1·6095	13	2·5649
1·66	0·5068	6	1·7919	14	2·6390
2	0·6931	7	1·9459	15	2·7081

6.—SMITHSONIAN TABLE (for Hygrometry).

Air Temp.	Difference between Wet and Dry Bulbs in ° C.										
	0	1	2	3	4	5	6	7	8	9	10
° C.	Saturation Pressures in mm. of Mercury.										
0	4·6	3·7	2·9	2·1	1·3						
2	5·3	4·4	3·6	2·7	1·9	1·1	0·3				
4	6·1	5·2	4·3	3·4	2·6	1·8	0·9				
6	7·0	6 0	5·1	4·2	3·3	2·4	1·6				
8	8·0	7·0	6·0	5·0	4·1	3·2	2·3	1·4	0·6		
10	9·2	8·1	7·0	6·0	5·0	4·0	3·1	2·2	1·3		
12	10·5	9·3	8·2	7·1	6·0	5·0	4·0	3·0	2·1	1·2	0·3
14	11·9	10·7	9·4	8·3	7·1	6·1	5·0	4·0	3·0	2·0	1·1
16	13·5	12·2	10·9	9·7	8·4	7·3	6·3	5·0	4·0	3·0	1·9
18	15·4	13·9	12·5	11·2	9·9	8·6	7·4	6·3	5·1	4·0	3·0
20	17·4	15·9	14·3	12·9	11·5	10·2	8·8	7·6	6·4	5·2	4·1
22	19·7	18 0	16·4	14·8	13·3	11·9	10·5	9·1	7·8	6·6	5·4
24	22·2	20·4	18·6	17·0	15·3	13·8	12·3	10·9	9·4	8·1	6.8

For intermediate temperatures take proportional values.

ANSWERS.

Exercises I.

(1) – 94° F., – 56° R.: 24·4° C., 19·5° R.: – 30° C., – 22° F.: 32° F., 0° R: 10° C., 8° R.: 20° C., 16° R.: 80° C., 176° F.: 197·6° F., 73·6° R.: – 10° C., – 8° R.: 62·5° C., 144·5° F.

(2) – 25·6°. **(3)** 131·3° F., 55·2° C.

(4) $35\frac{3}{5}$° F., $37\frac{2}{5}$° F., $39\frac{1}{5}$° F., 41° F., $42\frac{4}{5}$° F., $44\frac{3}{5}$° F., $46\frac{2}{5}$° F., $48\frac{1}{5}$° F., 50° F., $51\frac{4}{5}$° F., $53\frac{3}{5}$° F. (*whole* numbers on *both* = 41° F., 50° F.).

Exercises II.

(1) ·03 cm. **(2)** 72·2 cm. **(4)** 30·77° C. **(5)** 124·94° C.

(6) 100·008 cm. **(7)** ·0012 V. **(8)** ·000025; 99·85; 99·75.

(9) ·000007224 (nearly).

(10) V $(1 + 30c + 100c')$. $\dfrac{(30c + 100c')}{130\,(1 + 30c)}$ **(11)** $\dfrac{d_{100}}{d_{-100}} = \cdot 9833.$

(12) $(1 + ct)$; $(1 - c'\mathrm{T})$. **(13)** 10·15 grams.

(14) – ·00003225. **(15)** $\dfrac{\delta - \delta'}{\delta'\,(t' - t)}\,[t' > t]$.

(16) 193·87 grams. **(17)** 386·25° C. (nearly).

(18) ·06 inch approx. **(19)** 1·0015 pint.

(20) 1 c.c. of water becomes 1·105 c.c. of ice.

Exercises III.

(1) ·000017. **(2)** 9·04 grm. [C_a = ·000158]. **(3)** 100° C.

(4) ·000029. **(5)** 80 c.c.; 81·3347; ·000174 (nearly).

(6) 764·9462 mm. [C_r = ·0001815]. **(7)** ·00002783.

(8) 287·1. **(9)** 1·54 c.c. **(10)** 762·4 mm. **(11)** 56 m

(12) ·0000147. **(13)** 1·0543. **(14)** 98·6° C.

(15) 29·46 inches. **(16)** 100·15° C.

Exercises IV.

(1) 0·0038. **(2)** 663·77. **(3)** $\begin{cases} (1)\ \mathrm{P},\ \mathrm{T'},\ \dfrac{\mathrm{T'V}}{\mathrm{T}}. \\ (2)\ \dfrac{\mathrm{T'P}}{\mathrm{T}},\ \mathrm{T},\ \mathrm{V}. \end{cases}$

(4) $\frac{1}{C_r} = 273{\cdot}68$. **(5)** 19° C. **(6)** 76·23 cm. **(7)** 22·79 c.c.
(8) ·0841 gram. **(9)** $d_{10} : d_{15} = 1{\cdot}00428 : 1$. **(10)** 313° C.
(11) 273° C. **(12)** 93·75 atmospheres. **(13)** 534 c.c.
(14) 450·531 cub. in. **(15)** ·3471 gram increase.
(16) 195 : 112. **(17)** 587·7 cm. mercury.
(18) 496·8 k.g. and 261·8 k.g. **(19)** 28·586.
(20) 163° C. roughly.

Exercises V.

(1) 25·89° C.; 9·12 grm. **(2)** 12·1579 grm. **(3)** ·4614.
(4) ·0903 (nearly). **(5)** ·6153. **(6)** 13·21° C.
(7) 745·3° C. **(8)** 77·29° C. (nearly). **(9)** 1·005; 1·027; 1·016.
(10) $\frac{M(s't' + s''t'') + mst}{M(s' + s'') + ms}$. **(11)** 52·33° C. (nearly). **(12)** 3·41.

Exercises VI.

(1) 95·58 pound degrees. **(2)** − 3·6° C. **(3)** ·9047.
(4) 79·561. **(5)** ·1108. **(6)** ·0334. **(7)** ·9167.
(8) ·1098 (nearly). **(9)** 5·66 lb. **(10)** 80.
(11) 79·705. **(12)** 47·5 gm. degrees. 59·375 grams.

Exercises VII.

(1) 536·3. **(2)** 37·3. **(3)** 536·22. **(4)** 2,900 lb.
(5) 72·75 grm. **(6)** 564·98. **(7)** 65·3° C. **(8)** 90·34° C.
(9) 966·6; 144. **(11)** 506 mm. (approx.); 642,000; 10,654 ft.
(12) 96·25 approx. **(13)** 1,117·04 c.c. **(14)** ·0242 c.c.
(15) 320° C. **(16)** 2,024 calories. **(17)** 35·11° C.
(18) 1·978 grm. **(19)** 6·7 grm.
(20) $k = {\cdot}2116$ (taking the hour as unit of time); ·8224 grm. per hour

Exercises VIII.

(1) 1·205 grm. **(2)** − 5·2° C. **(3)** 10·1° C.; 77·1 per cent.
(4) $h = 72{\cdot}76$ per cent. **(5)** ·0166 grm.
(6) 46 per cent.; ·0079 gram. **(7)** 12·366 grams. **(8)** 90·6 c.c.
(9) 358 grm. weight. **(10)** 68; 1·213 grm.
(11) 23·09° C.; ·146. **(12)** 2·6025 litres.

Exercises IX.

(1) ·0384. **(2)** 12,000,000 calories. **(3)** 125·28 calories.
(4) 4,312,500 calories. **(5)** ·0672. **(6)** ·00013. **(7)** 1,800.
(8) See worked example No. 9. **(9)** See worked example No. 9.
(10) $\frac{1}{512}$; Coeff. of trans. = $\frac{1}{1024}$. **(11)** ·04 calorie. **(12)** ·0005.

Exercises X.

(1) ·084.
(2) 29272·7 cm.-grm. (the actual figures will depend on the data assumed in the solution). **(4)** ·1683; ·2373. **(5)** 96·6.
(6) 5·7. **(7)** 1125·5 B.Th.U.; 12·78 lb. (taking 15,000 for coal.)
(8) 80 per cent. **(9)** 17,960 B.Th.U.
(10) 16,490 B.Th.U.; 15·4 per cent.
(11) ·15° C.; the difference ·1° C. is due to internal work.
(12) 289·83.

Exercises XI.

(1) 1,467 ft.-pounds: 1·05 pound degree C. units.
(2) ·635 c.m.: − 109·5° C.: 4,602 kg.m. **(3)** ·26.
(4) ·21; 413·2. **(5)** ·178. **(6)** 1,115. **(7)** 73·45 : 1.
(8) ·28° C. **(9)** About ·31 lb. **(10)** 153 atmospheres.
(11) 2·78° C. **(12)** 8·528 (ft.-lb. C.). **(13)** About 4·6 times as great.
(14) 25·6 atmospheres: 425° C.; 80,542 ft.-pds. **(15)** 82 per cent.
(16) About 12,300.

Exercises XII.

(1) 13,925 B.Th.U.; 10·95 lb.; about 22 lb. **(2)** ·5 inch.
(3) 52·6. **(4)** 61,420 ft.-pds; 223·4; 178·7.
(5) 1·07″; 74·9; 240·3; 5·82. **(7)** 121·7.
(8) 10·9 B.Th.U.; 16·7 %. **(9)** 11·52; H.P. 1618, I.P. 1672, L.P. 1658.
(10) Mech. Eff. = 76 per cent.; Heat Eff. (on I.H.P.) = 16·25 per cent.; on B.H.P. = 12·33 per cent. **(11)** ·245; ·418.
(12) 27/26 = 1·04. **(13)** 91 %; 4·43 %; 31·2 %.
(14) T.E. = 7,740 lb.; I.H.P. = 824.
(15) Cylinder diameters = 9·5″, 4·75″; B.H.P. = 21·5; T = 651.
(16) 30°; − 6·5°; 126°. **(17)** 4236.
(18) Centifrugal force ·003408 n^2r lb.; 242; 246.
(19) 141·4 lb. **(20)** 24·4 lb.

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